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Investigations Into Methods for Enhanced Damping Coefficient Separability and Shock Classification

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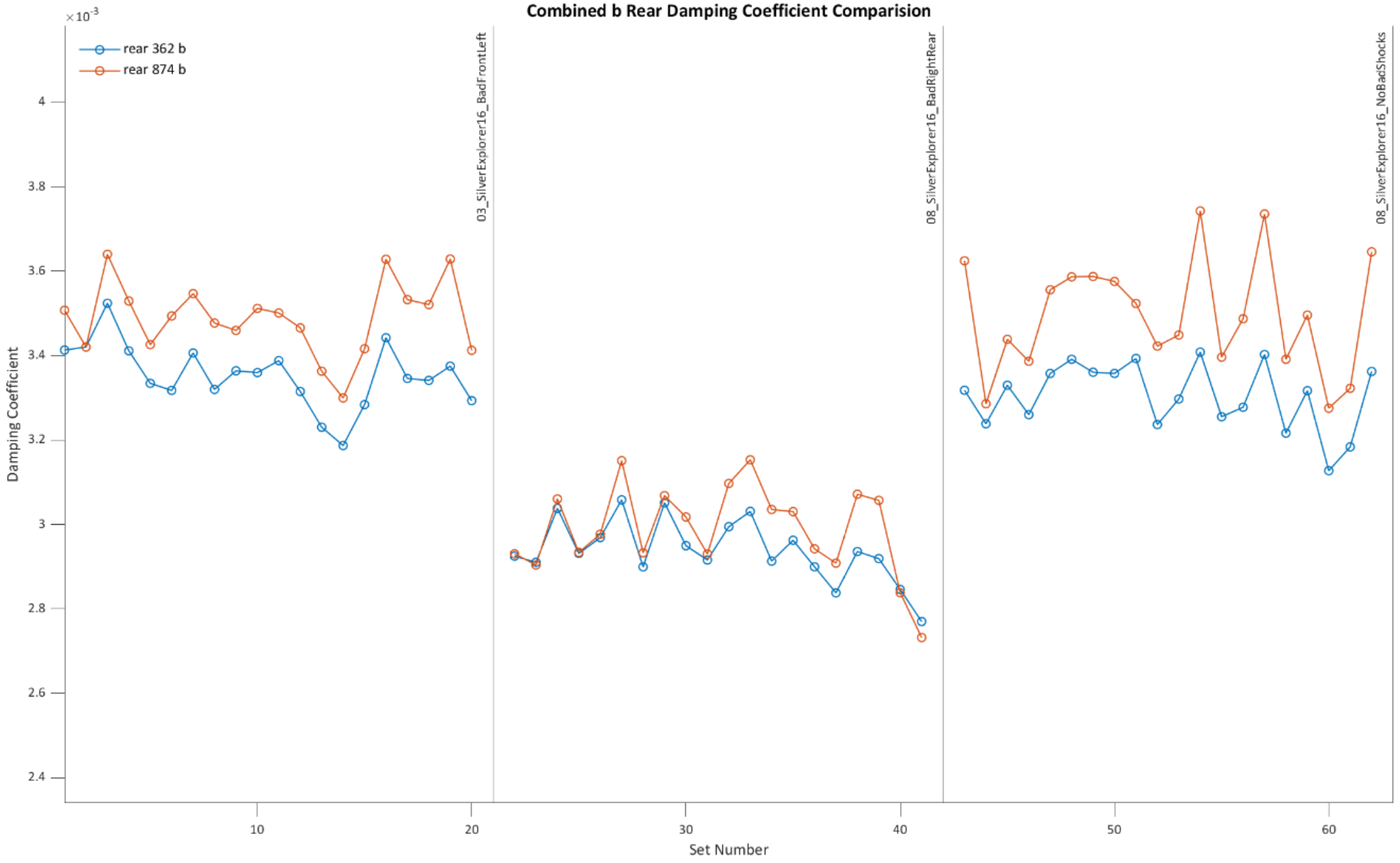
The Problem

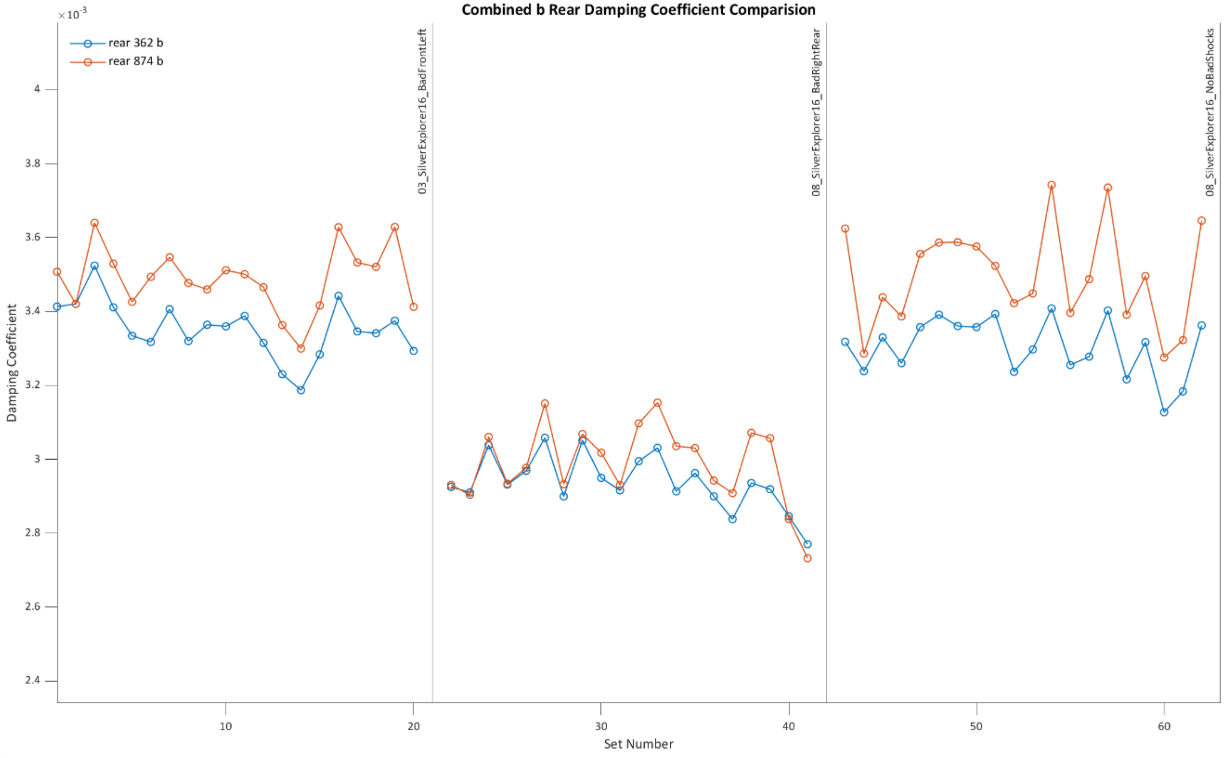
- **Existing Explorer damping coefficient variance is on the order of separability between good and 25% damping loss shock cases which makes thresholding unreliable**



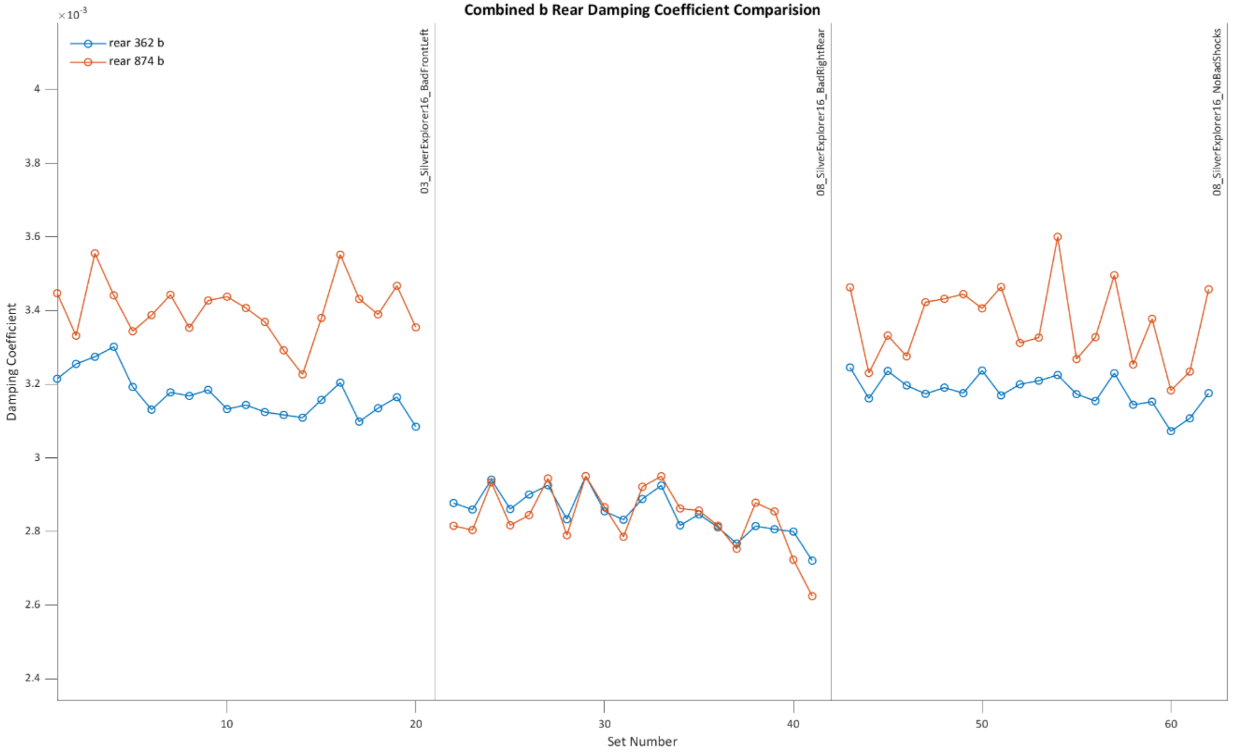
Current Correction Method:
Camera Undistortion

Current Correction and Classification Methods: Undistortion

- **Camera undistortion assists in separability with rear shock cases.**



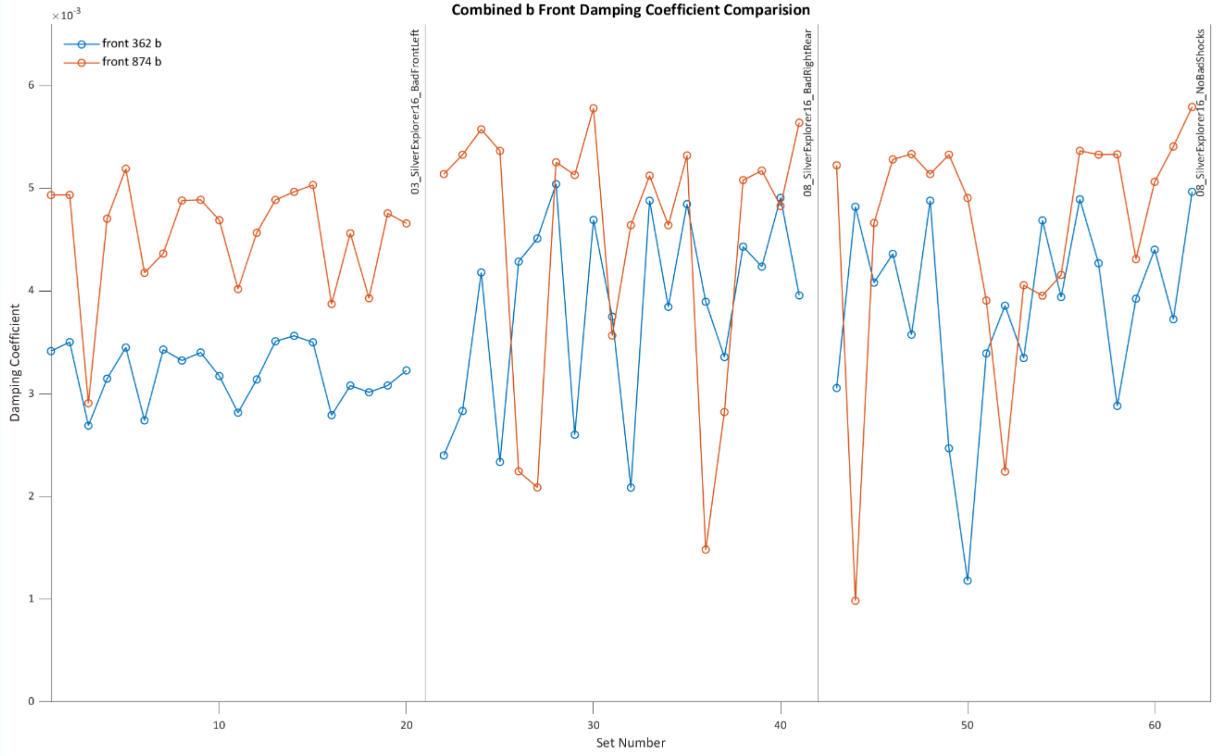
Distorted



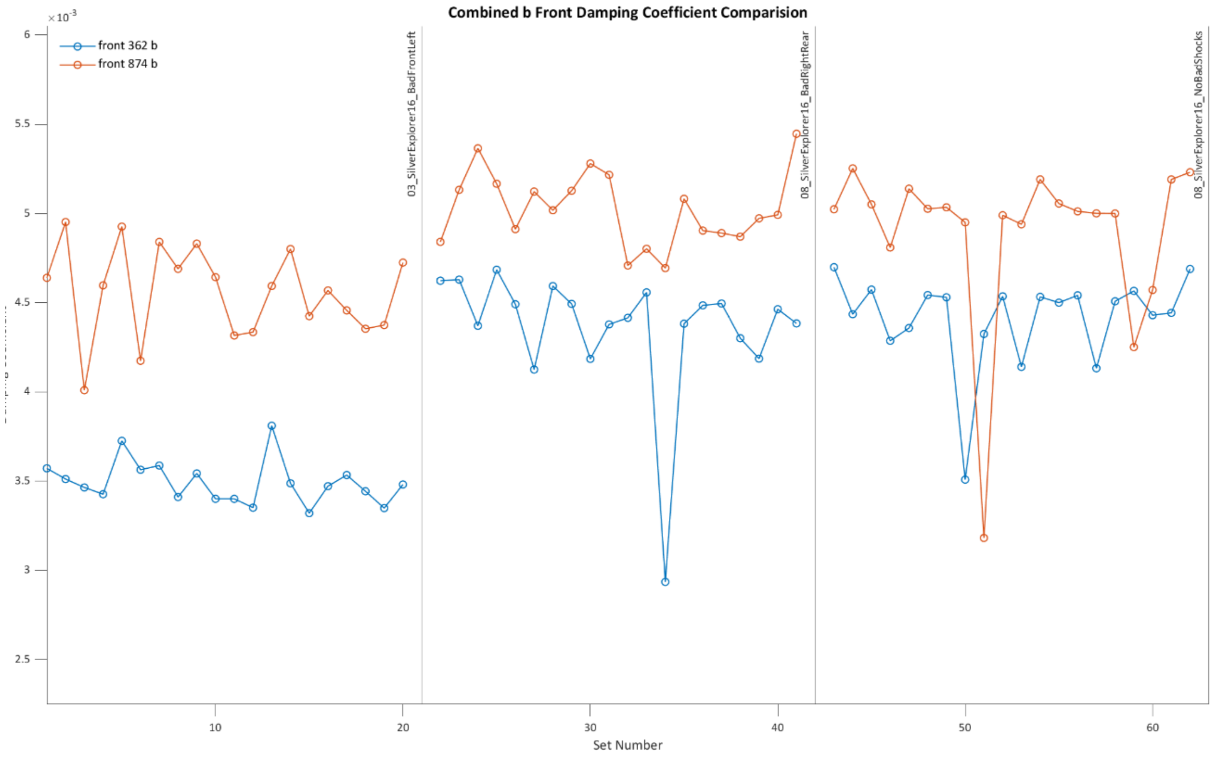
Undistorted

Current Correction and Classification Methods: Undistortion

- **Significant improvement in front damping coefficient variance**



Distorted

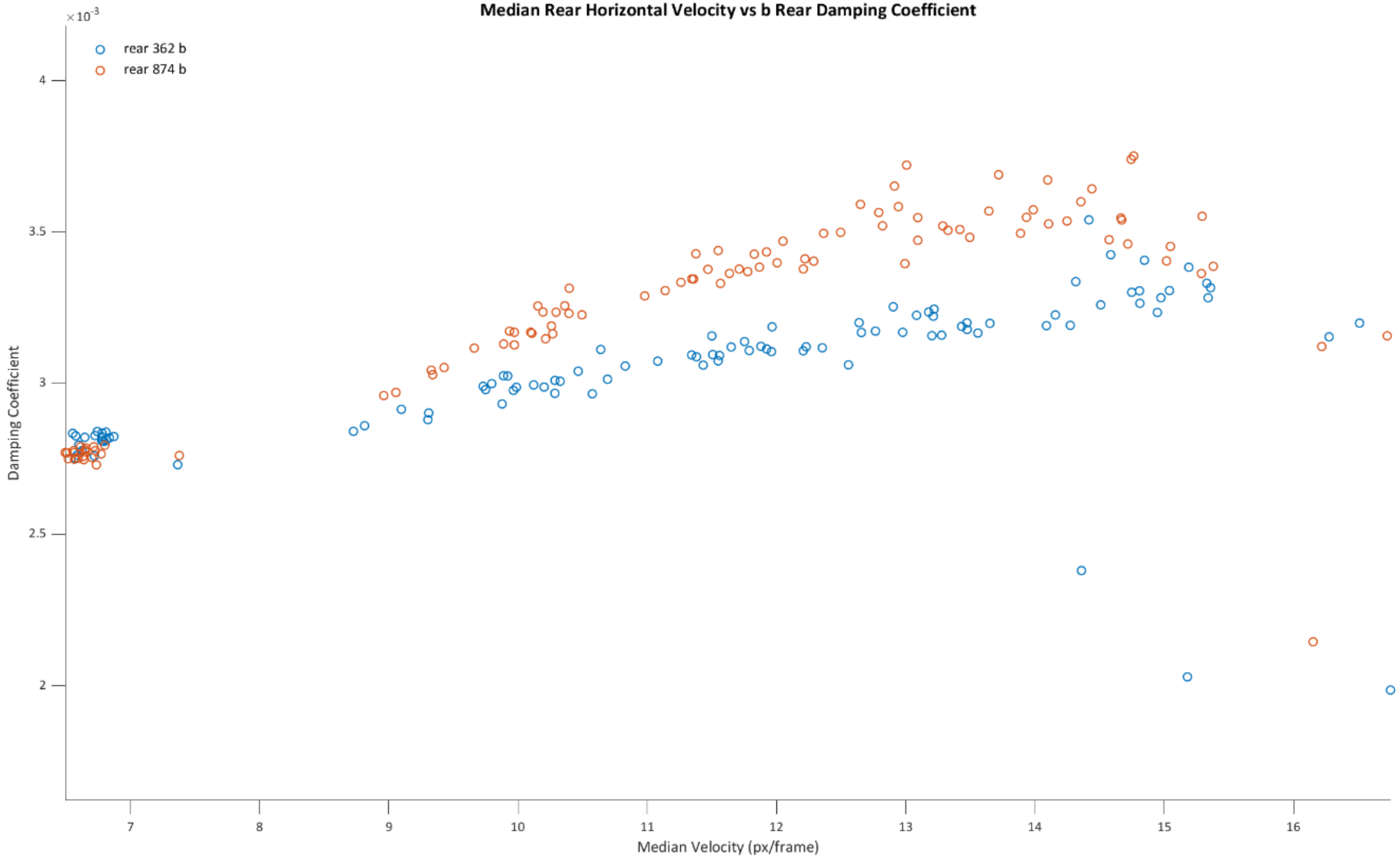


Undistorted

Current Correction Method: Horizontal Velocity

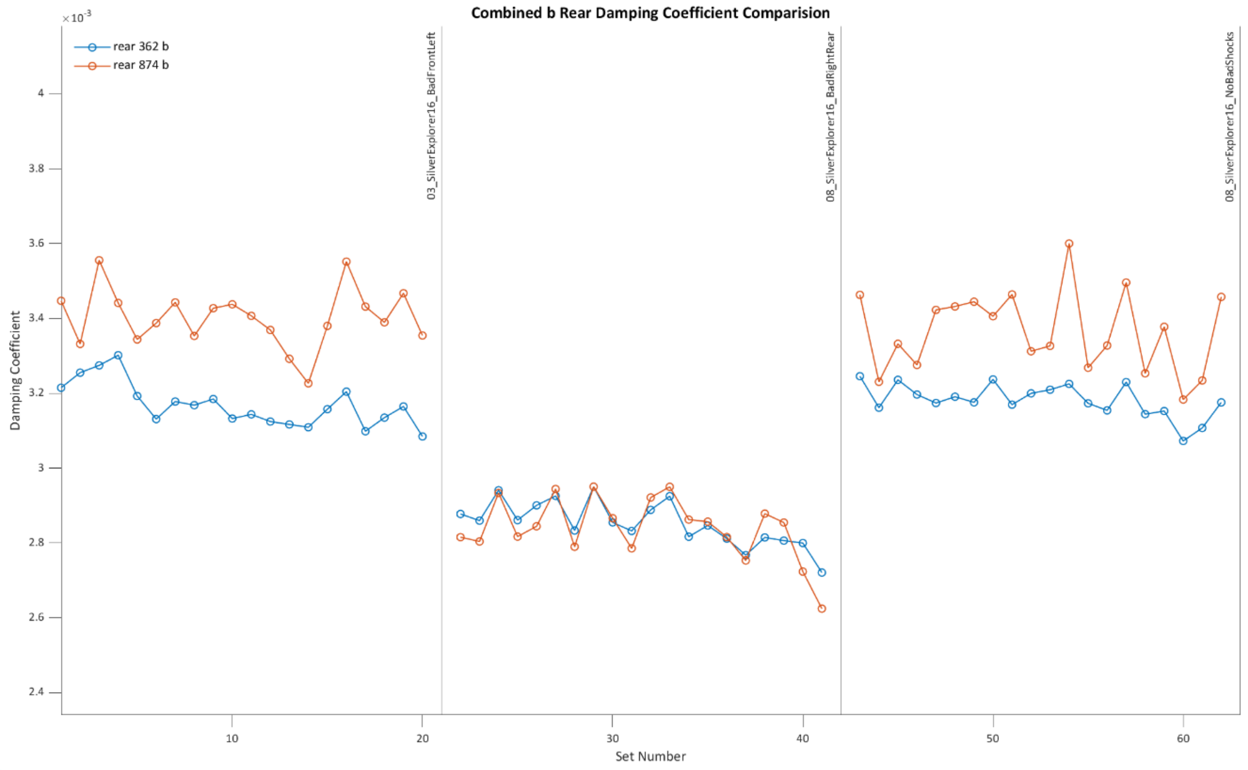
Current Correction and Classification Methods: Velocity

- **Rear damping coefficients have linear correlation with horizontal velocity**

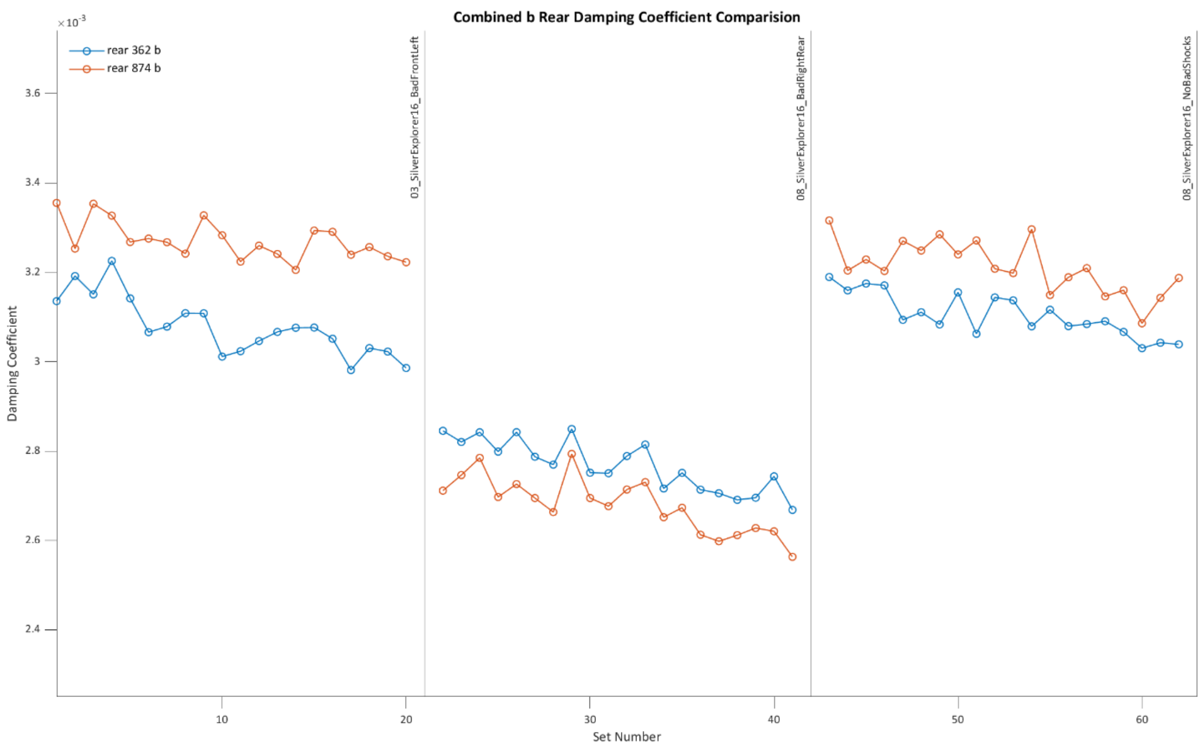


Current Correction and Classification Methods: Velocity

- **Linear velocity normalization reduces velocity-induced variance in rear damping coefficients**
- **Linear correction does not work for front shocks**



Uncorrected Rear

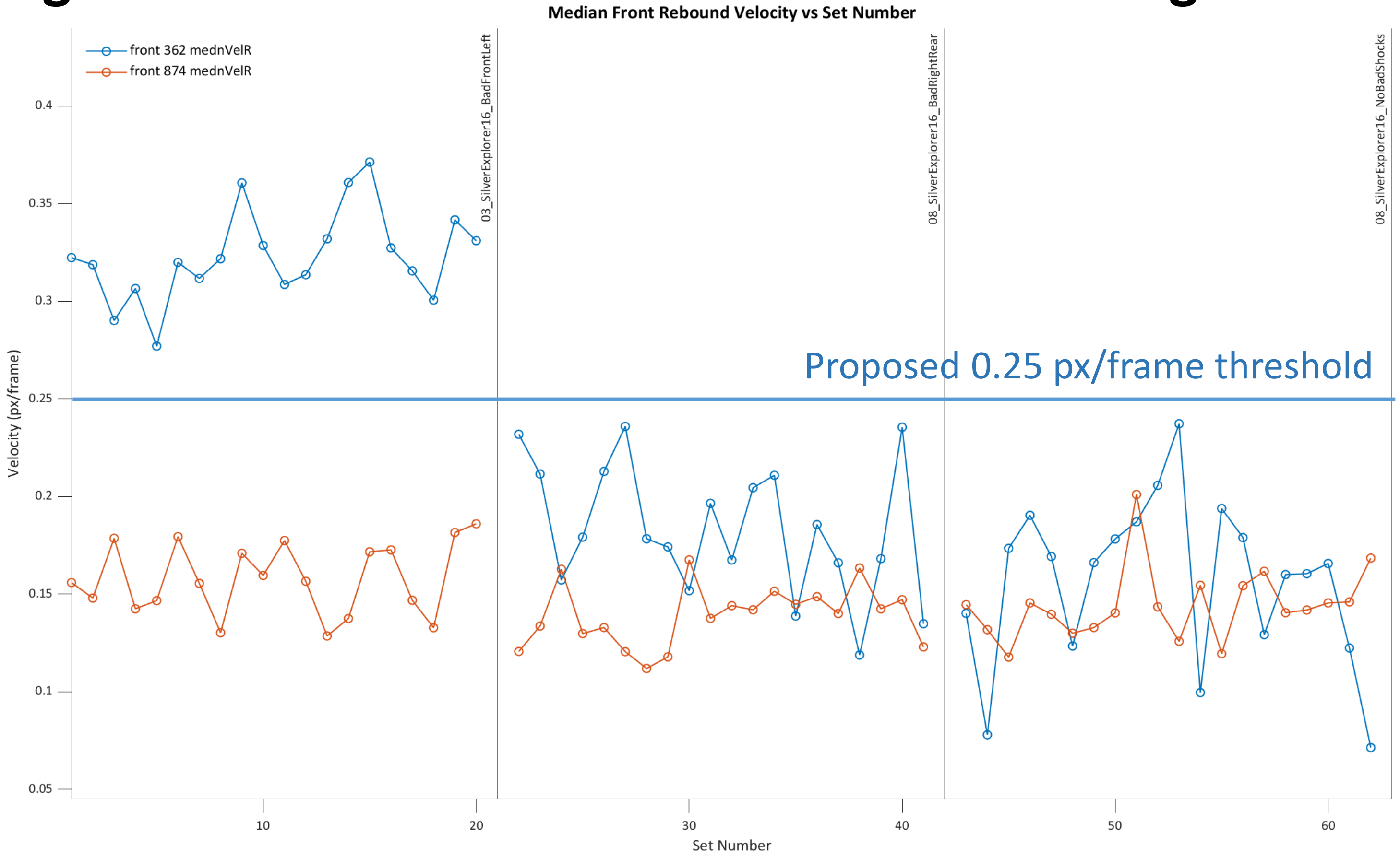


Corrected Rear

Classification Method:
Rebound Velocity

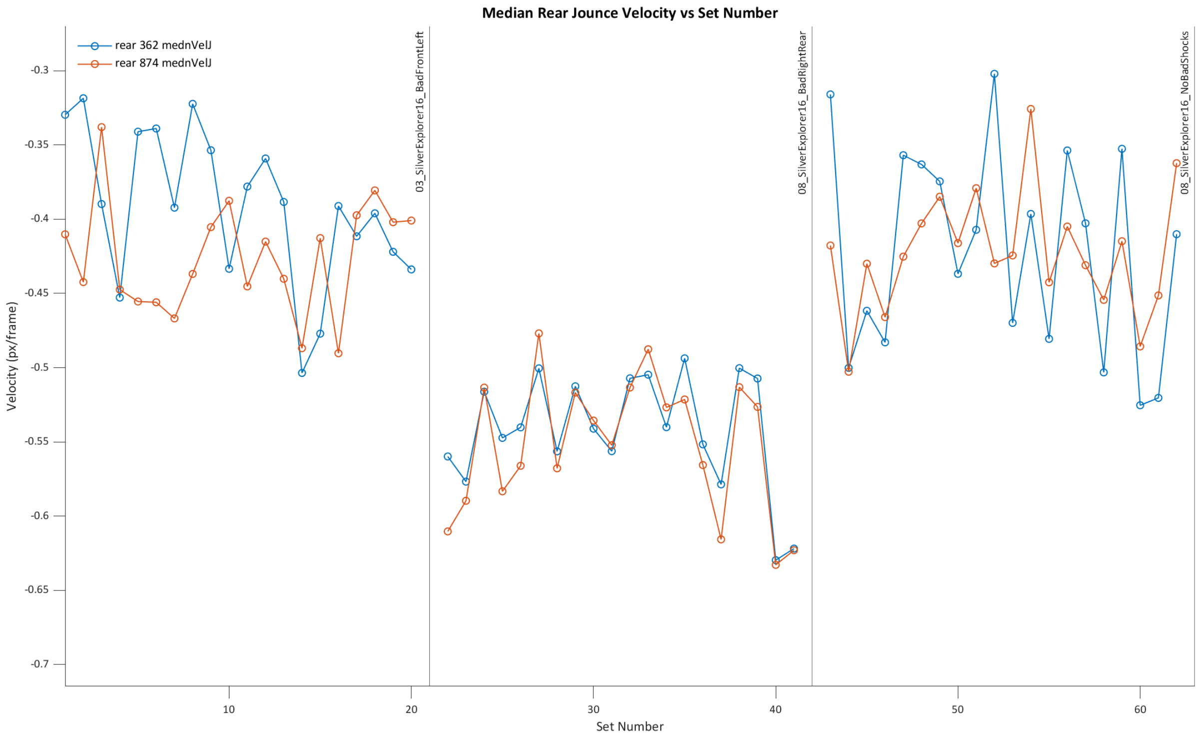
Alternative Classification Methods: Rebound Velocity

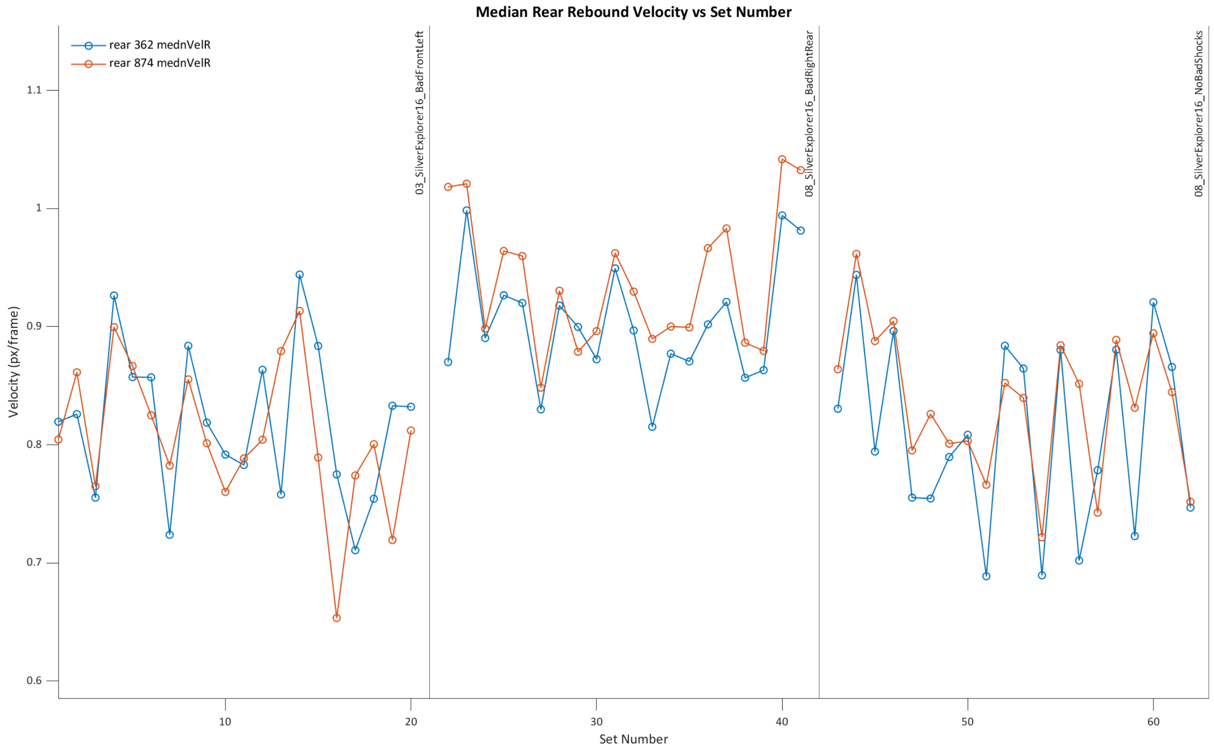
- **Front left rebound velocities for 25% damping loss shock cases were observed to a have a significantly higher median rebound velocity**
- **Need more testing to determine if same holds true for front right**



Alternative Classification Methods: Rebound Velocity

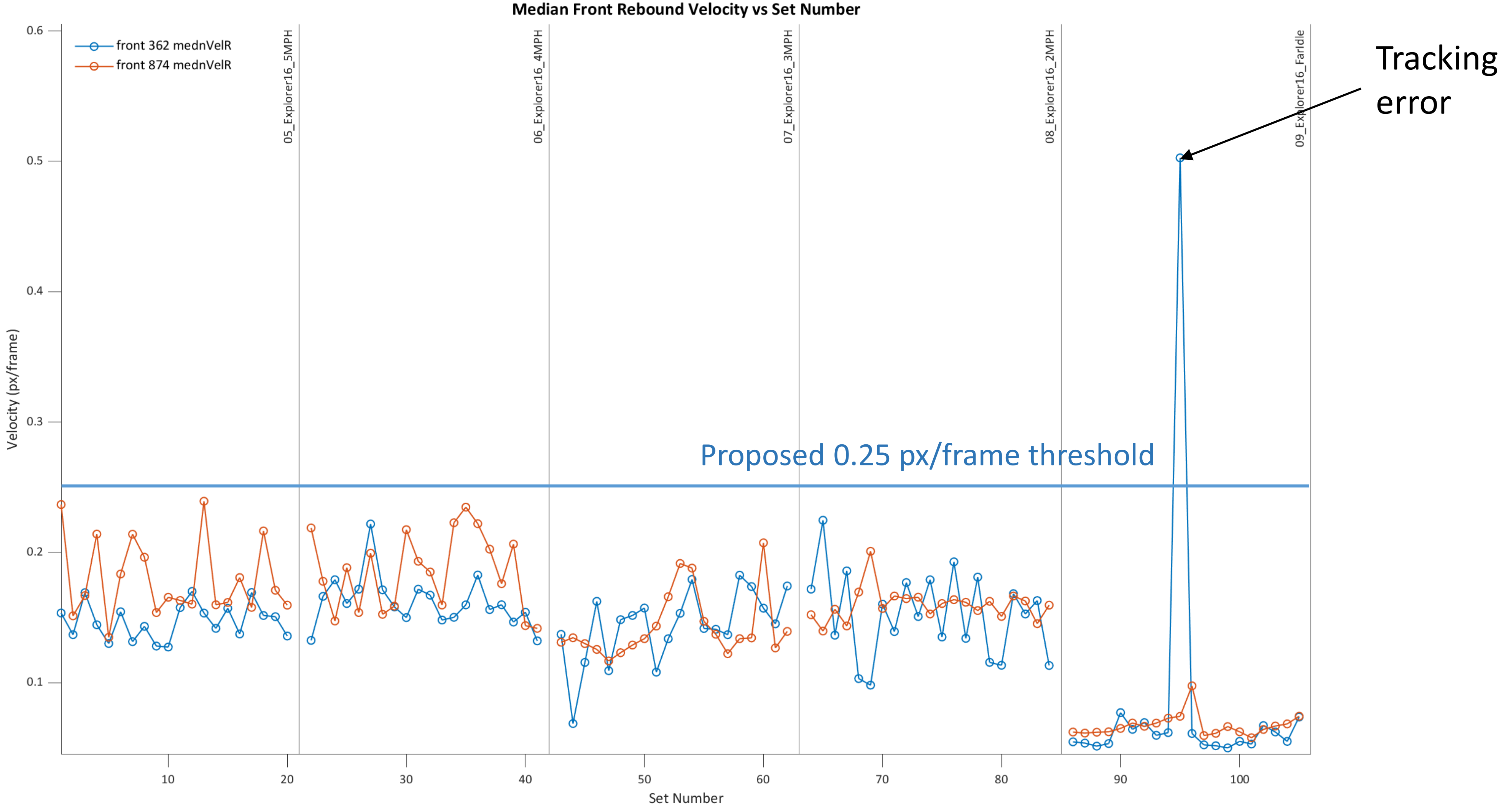
- Rear shocks have much more variance
- *May* be able to use compression velocity unreliably





Alternative Classification Methods: Rebound Velocity

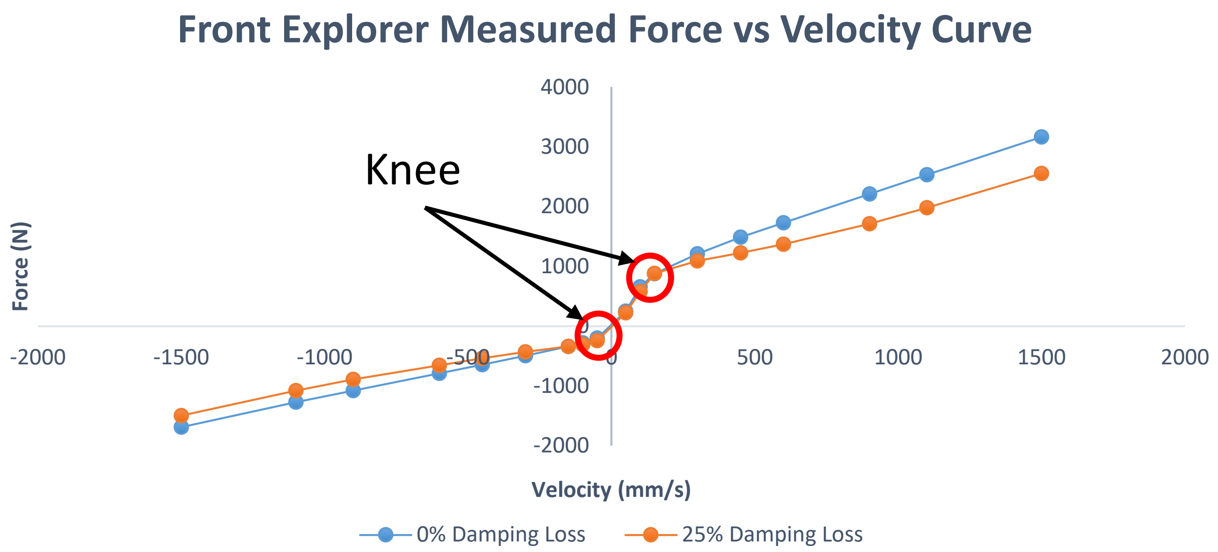
- **Very little-no dependence on horizontal vehicle velocity for front rebound velocity**

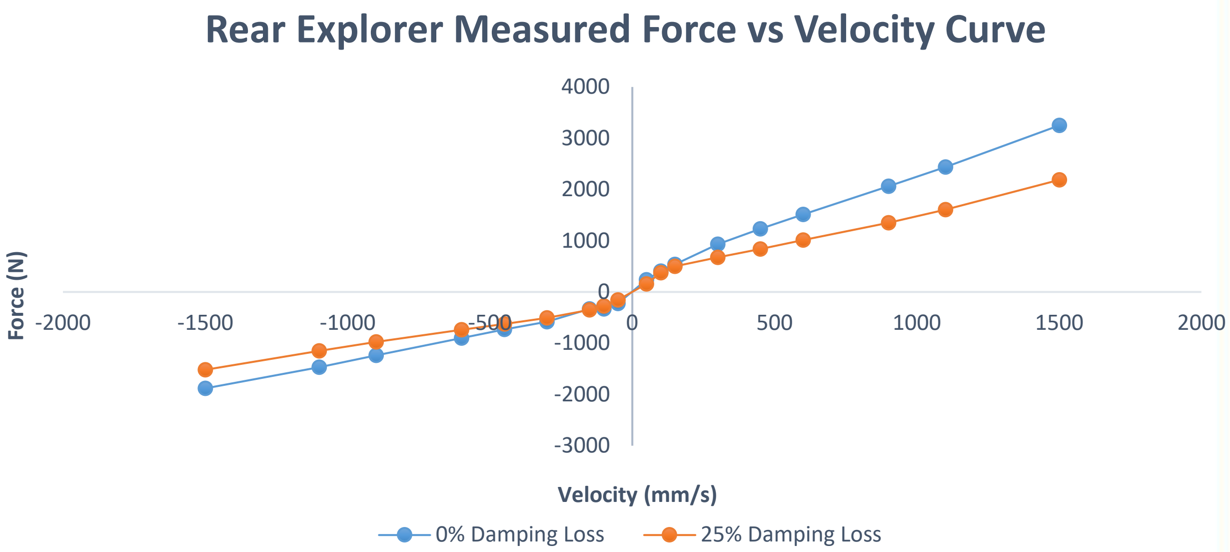


Classification Method: Damping Acceleration

Alternative Classification Methods: Damping Acceleration

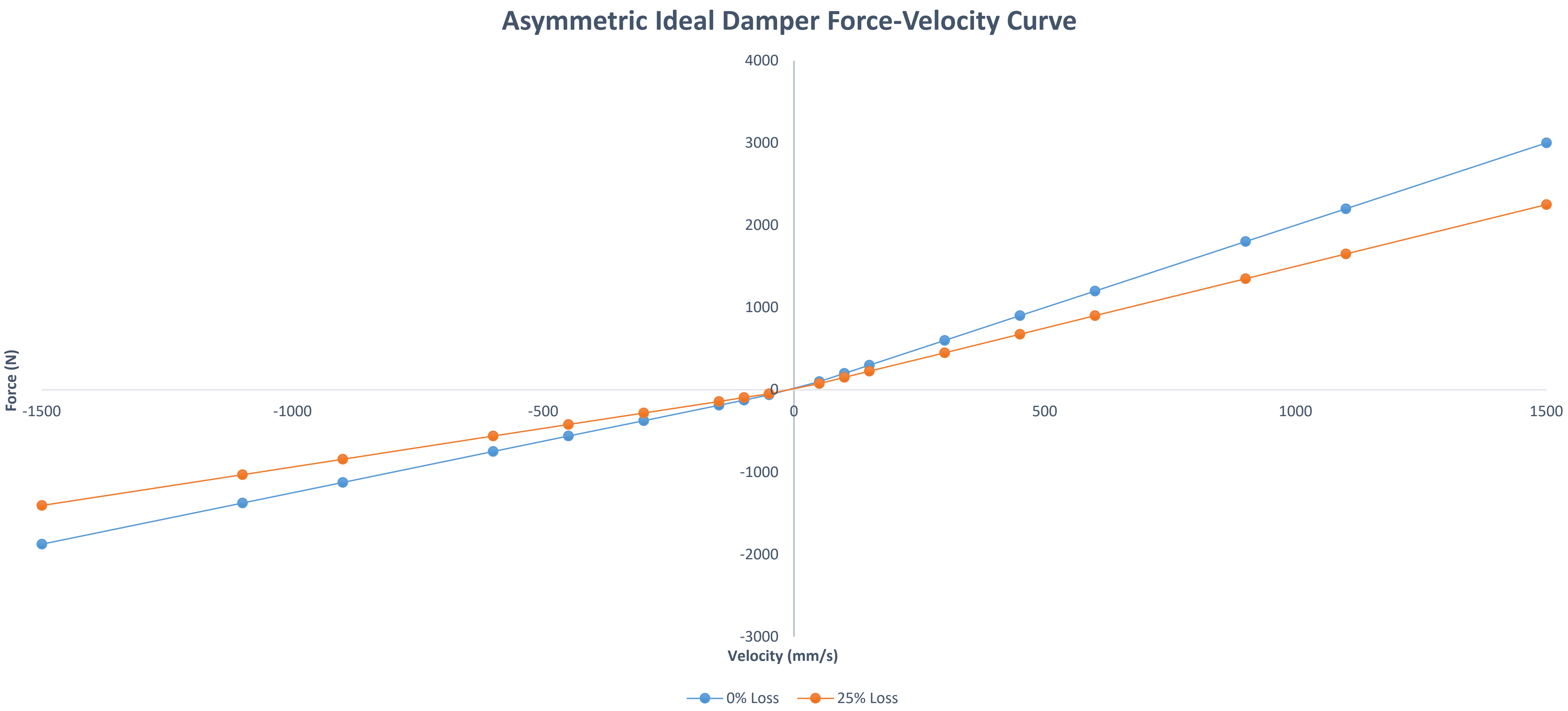
- **A critical document during shock absorber selection are the force-velocity curves.**
- **Measured force velocity curves for the Explorer:**





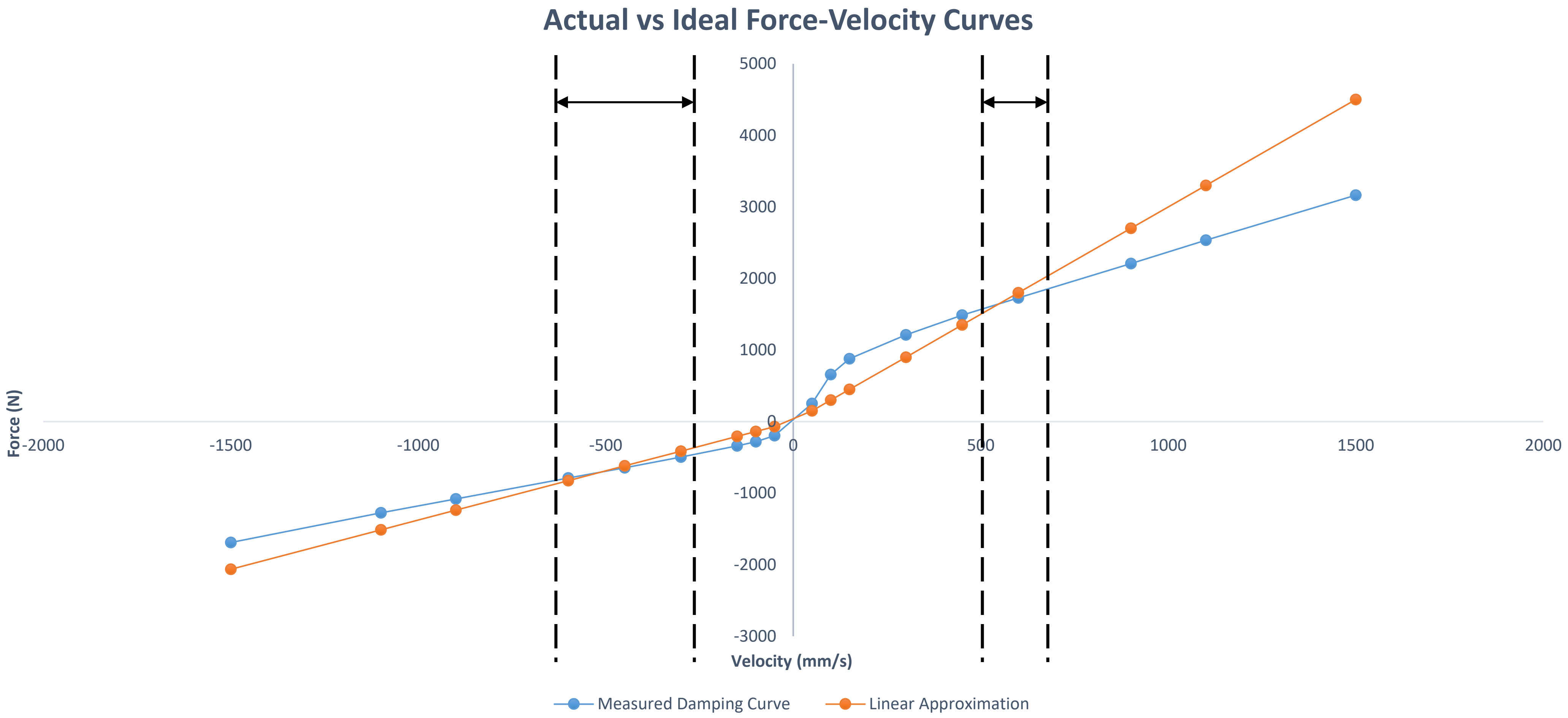
Alternative Classification Methods: Damping Acceleration

- **Current fitting functions assume asymmetric ideal force-velocity curve for dampers:**



Alternative Classification Methods: Damping Acceleration

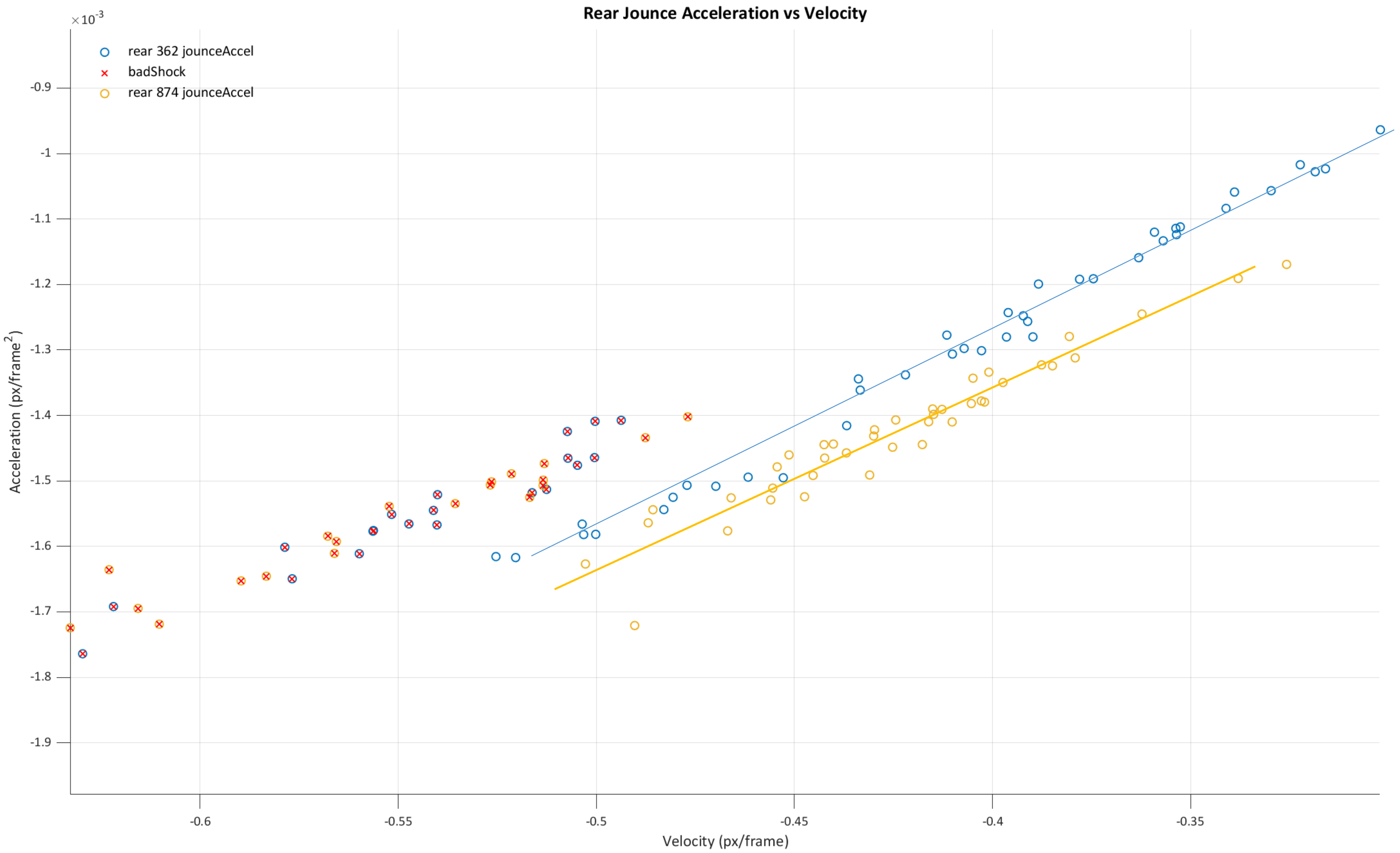
- **Theory: Fitted damping coefficient is linear approximation of damping curve at specific compression/rebound velocity ranges**



Alternative Classification Methods: Damping Acceleration

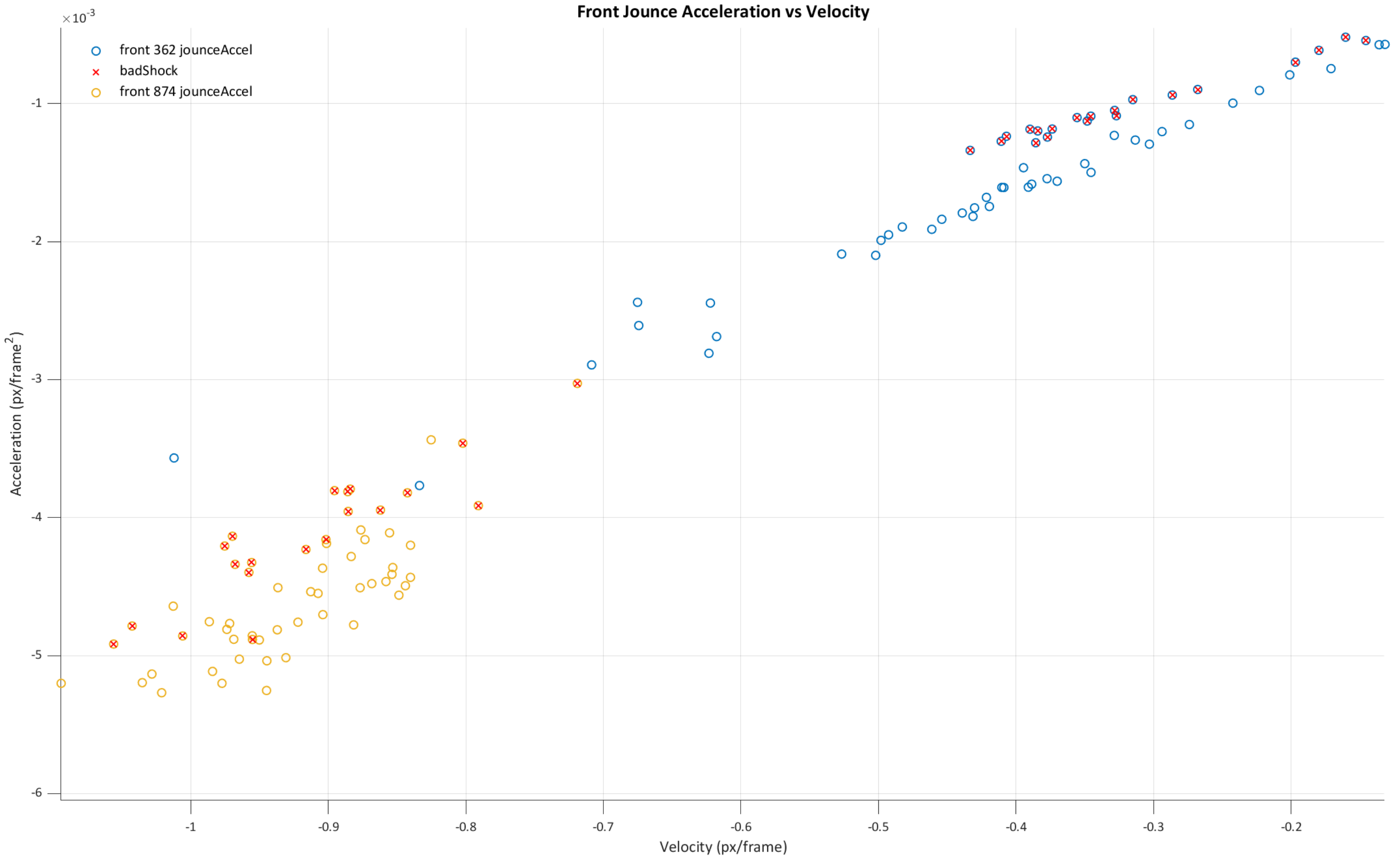
- **Multiplying the fitted damping coefficient by the median vertical velocity could recreate a velocity-acceleration curve.**
  - Acceleration and not velocity because physically the mathematic damping coefficient is the mechanical damping coefficient divided by mass
- **By calculating a line of best fit, we can threshold good vs. bad shocks based on how far away they are from the line of best fit**
- **No need for velocity correction as it is “built-in” to classification method**
- **Shortcoming is that we are assuming the velocity of the body is proportional to the velocity of the shock**
  - Ignoring tire dynamics
  - Ignoring bump
  - Ignoring road inconsistencies

Alternative Classification Methods: Damping Acceleration



Alternative Classification Methods: Damping Acceleration

- **Due to the nature of the front fits, the coefficients aren't accurate and therefore don't produce clean graphs like the rears**



Alternative Classification Methods: Damping Acceleration Results

- **Rear Shock Classification Statistics:**
  - Sensitivity: 100%
  - Specificity: 100%
- **Front Shock Classification Statistics***
  - Sensitivity: 90%
  - Specificity: 92.5%
- **Stronger linear correlation than horizontal velocity vs rear damping coefficient for current data**

Fitting Method: Dual Frequency

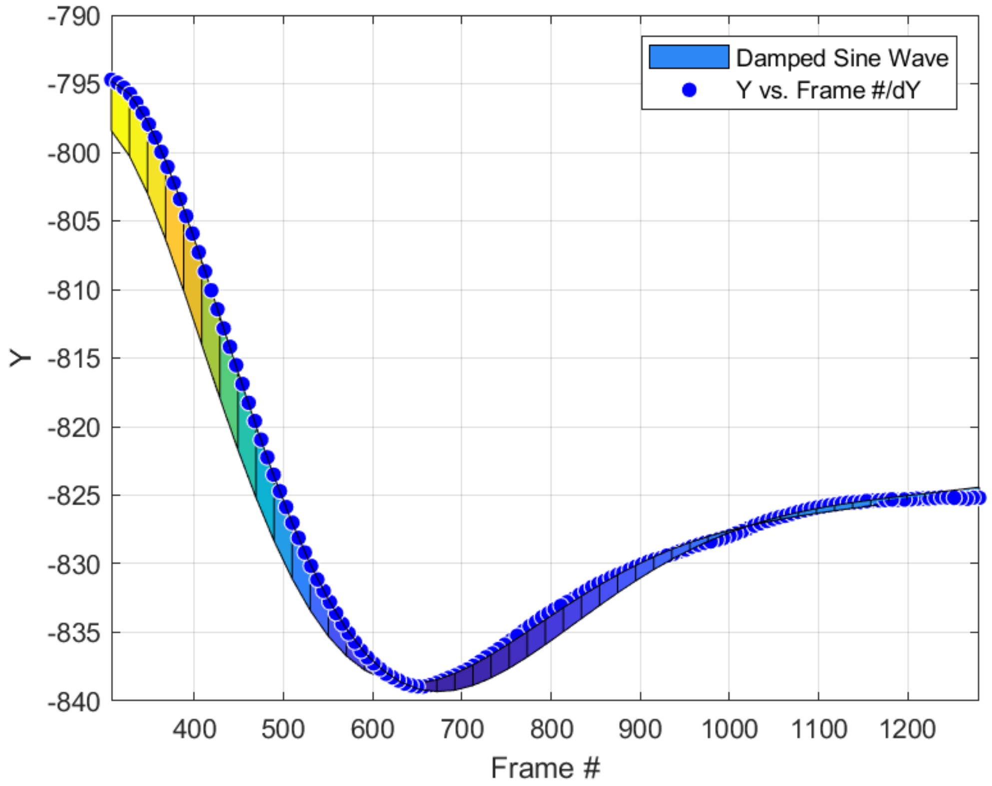
Alternative Fitting Function: Dual Frequency

- **Current fitting function accounts for non-symmetry of force-velocity curves through separate compression/rebound damping coefficients:**

$$y(t) = \begin{cases} Ae^{-at}\cos(\omega t + \theta) & dy \geq 0 \\ Ae^{-bt}\cos(\omega t + \theta) & dy < 0 \end{cases}$$

- **Proposed fitting function would account for non-symmetry of force-velocity curves through both separate damping coefficients and vibration frequencies:**

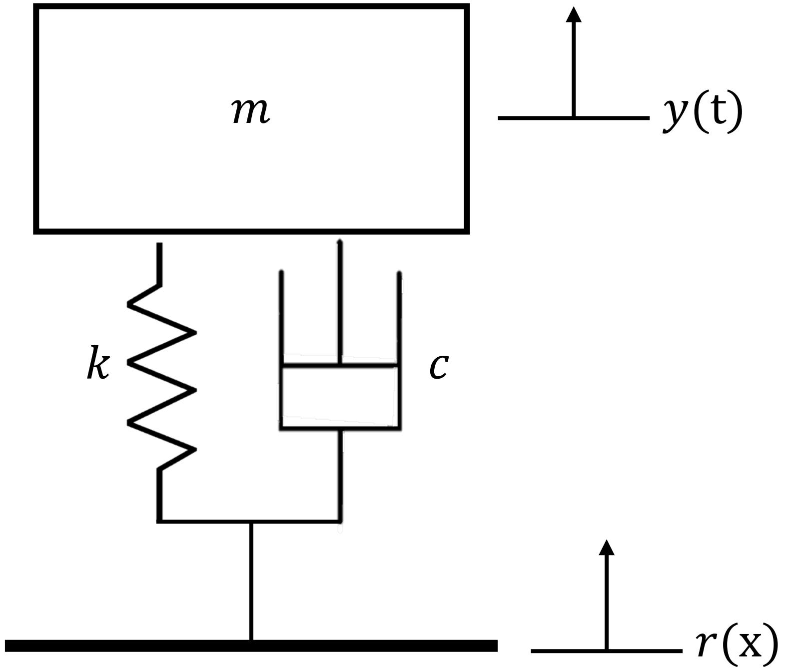
$$y(t) = \begin{cases} Ae^{-at}\cos(\omega_a t + \theta) & dy \geq 0 \\ Ae^{-bt}\cos(\omega_b t + \theta) & dy < 0 \end{cases}$$



Dual Frequency Damped Cosine

Theory Review: Idealized Mass-Spring-Damper System

Simplified Quarter-Car Suspension Model



$$m\ddot{y} + c\dot{y} + ky = c\dot{r} + kr$$

Homogenous Solution

$$m\ddot{y} + c\dot{y} + ky = 0$$

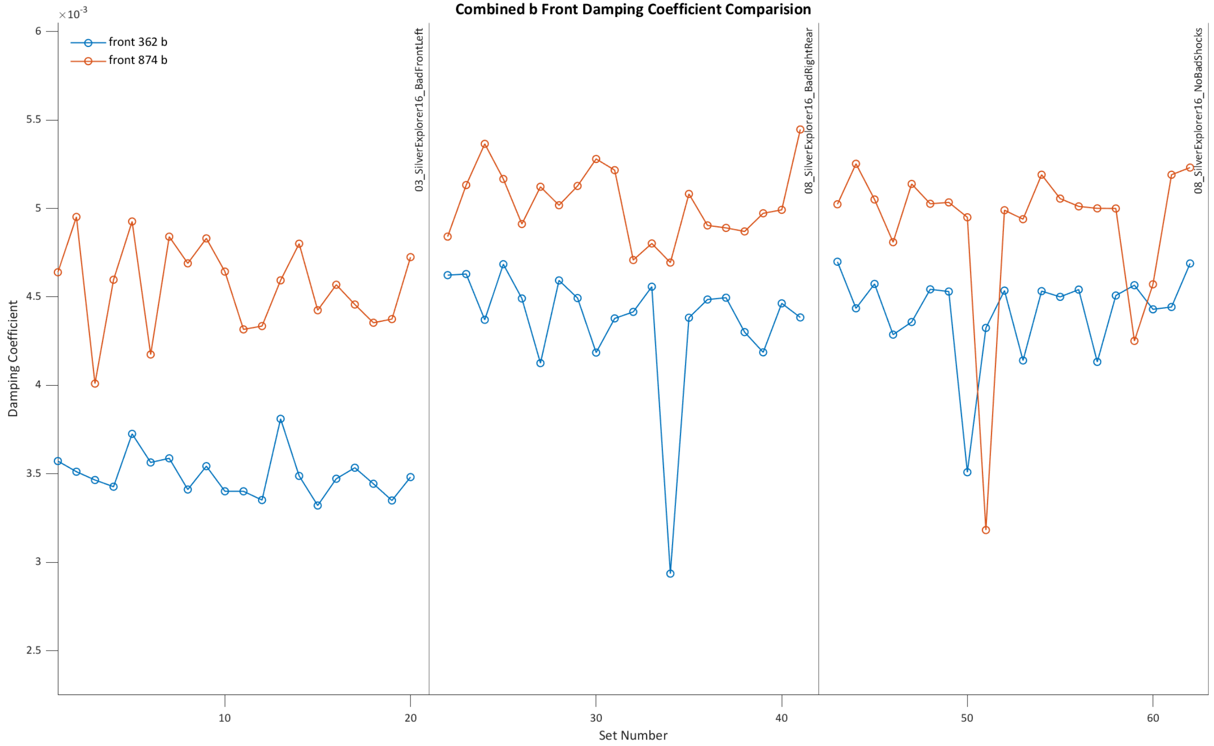
$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

Particular Solution (dependent on road profile and vehicle speed)

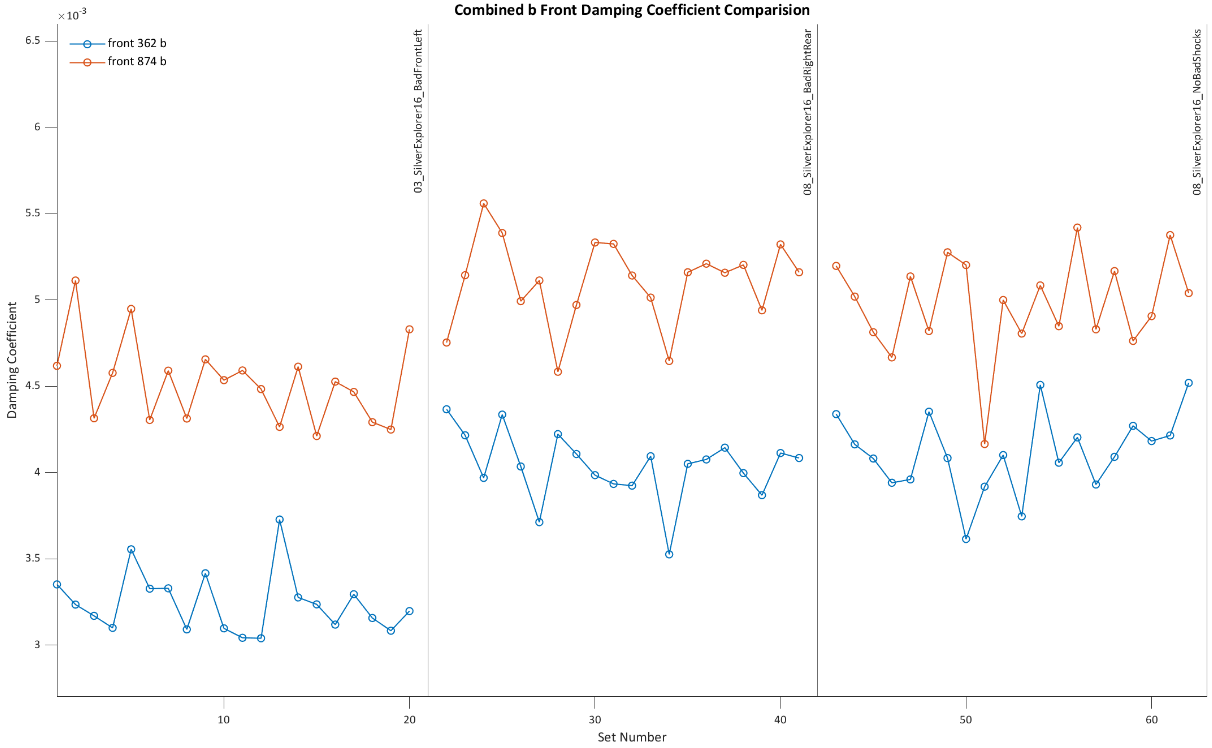
- Overdamped: $y(t) = Ae^{\frac{-c+\sqrt{c^2-4mk}}{2m}t} + Be^{\frac{-c-\sqrt{c^2-4mk}}{2m}t}$
- Critically Damped: $y(t) = Ae^{\frac{-c}{2m}t} + Bte^{\frac{-c}{2m}t}$
- Underdamped: $y(t) = Ae^{\frac{-c}{2m}t}\cos\left(\frac{\sqrt{4mk - c^2}}{2m}t - \theta\right)$

Alternative Fitting Function: Dual Frequency

- **Improvement in front damping coefficient variance**



Single Frequency



Dual Frequency

Alternative Fitting Function: Dual Frequency

- **Front Left Shock Classification Statistics:**
  - 3 False Positive
  - 1 False Negative
  - Sensitivity: 95%
  - Specificity: 92.5%
- **Better than using damping acceleration classification method**
- **Worse than using rebound velocity**
- **Very little change in rear shocks due to stronger inherent symmetry**

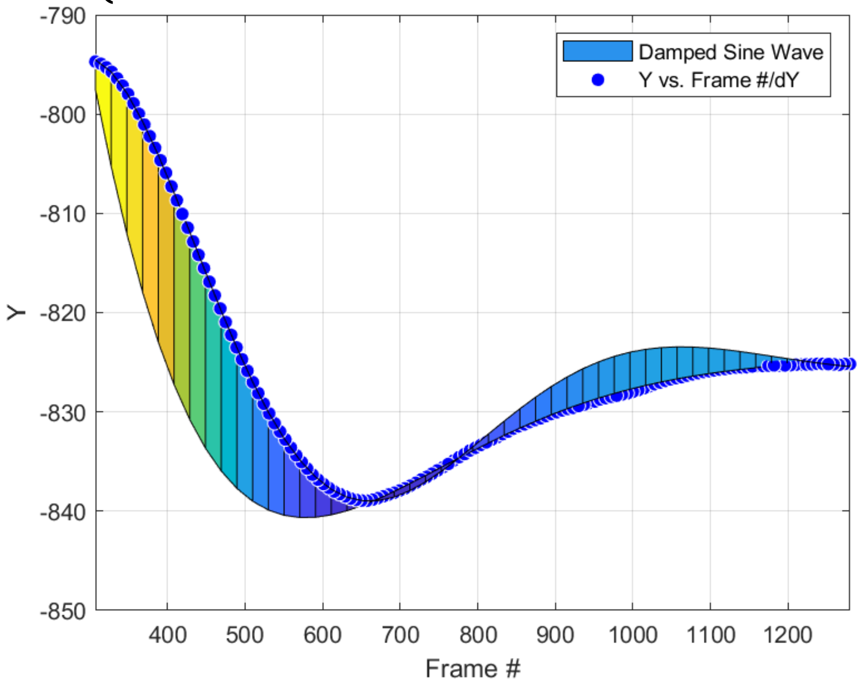
Fitting Method:

Completely Dissociated Underdamped Compression/Rebound

Alternative Fitting Function: Completely Dissociated Compression/Rebound

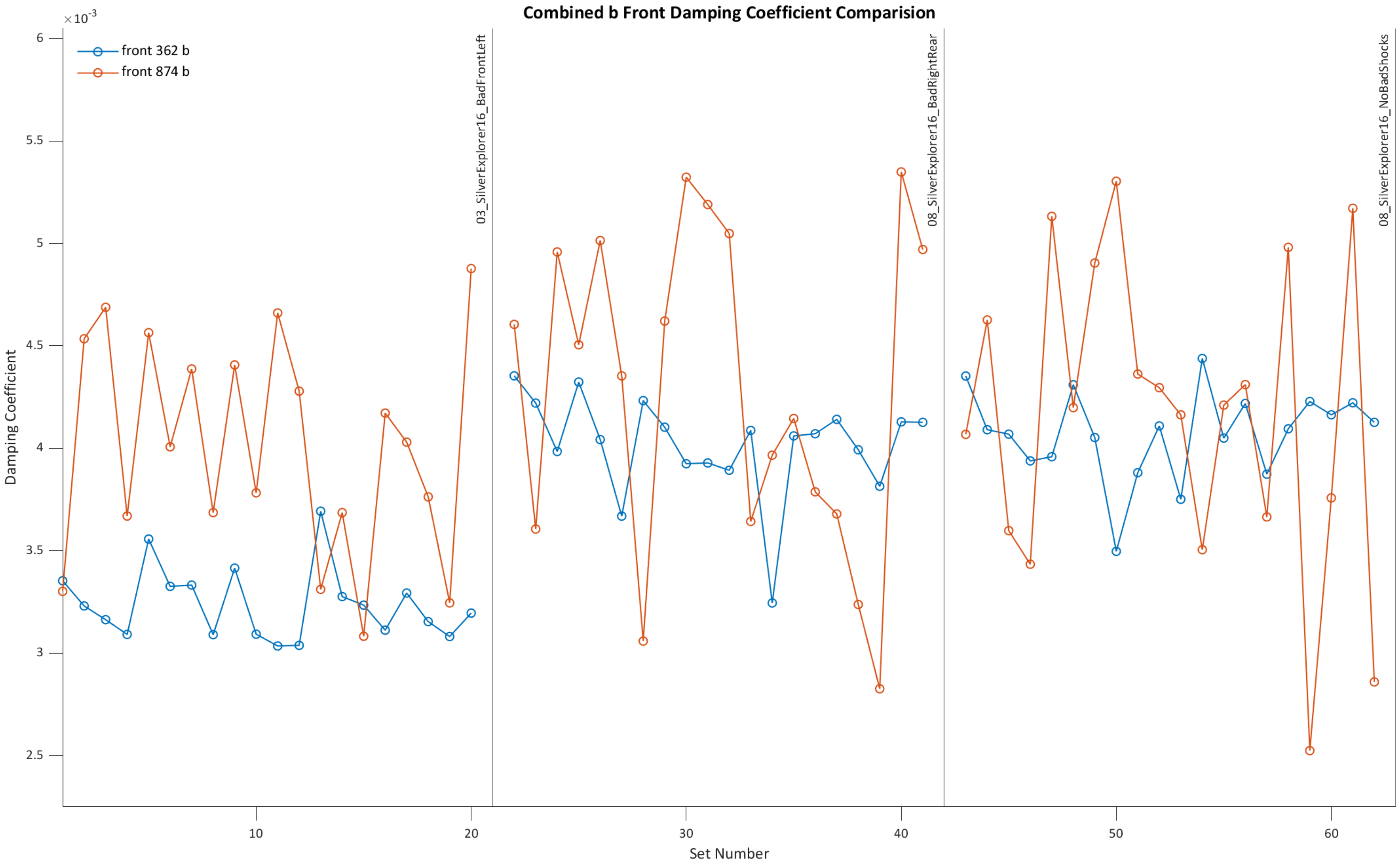
- **Damped cosine amplitude and phase angle are dependent on initial conditions as well as system parameters**
- **Proposed fitting function would separate all coefficients based on compression/rebound:**

$$y(t) = \begin{cases} A_a e^{-at} \cos(\omega_a t + \theta_a) & dy \geq 0 \\ A_b e^{-bt} \cos(\omega_b t + \theta_b) & dy < 0 \end{cases}$$



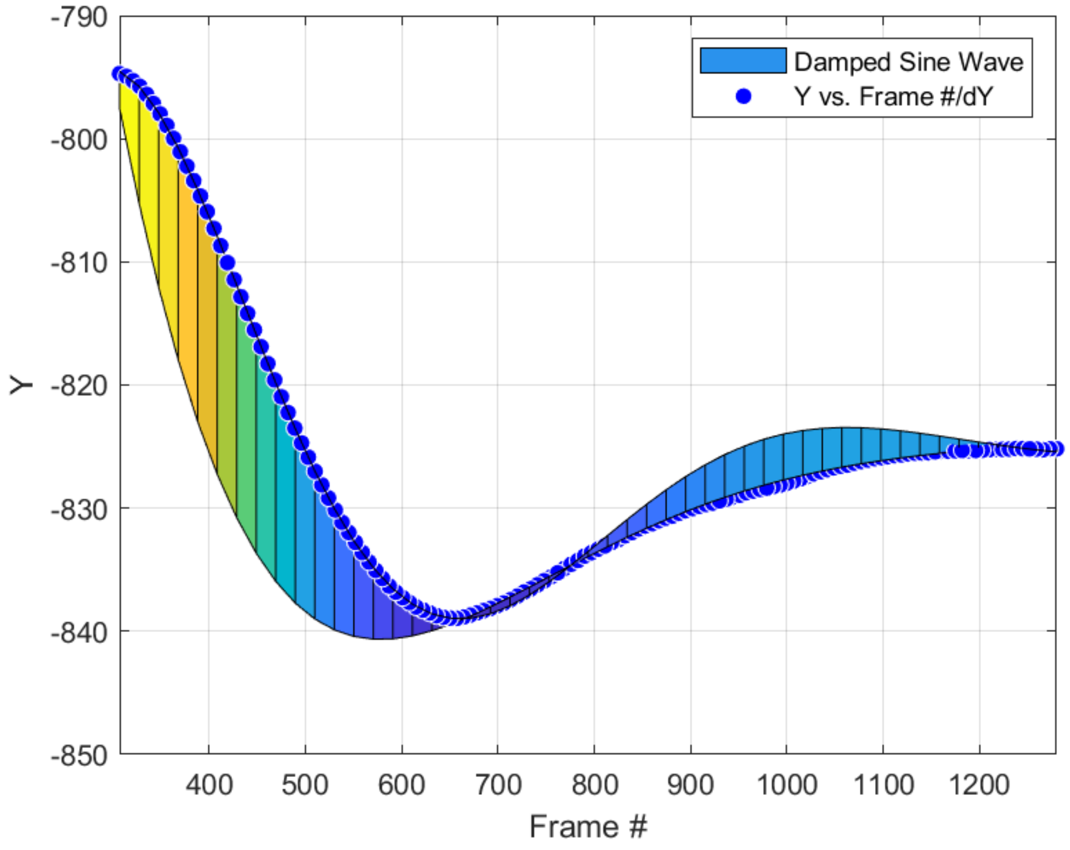
Alternative Fitting Function: Completely Dissociated Compression/Rebound

- **Fit uniqueness becomes an issue at this level of dissociation:**

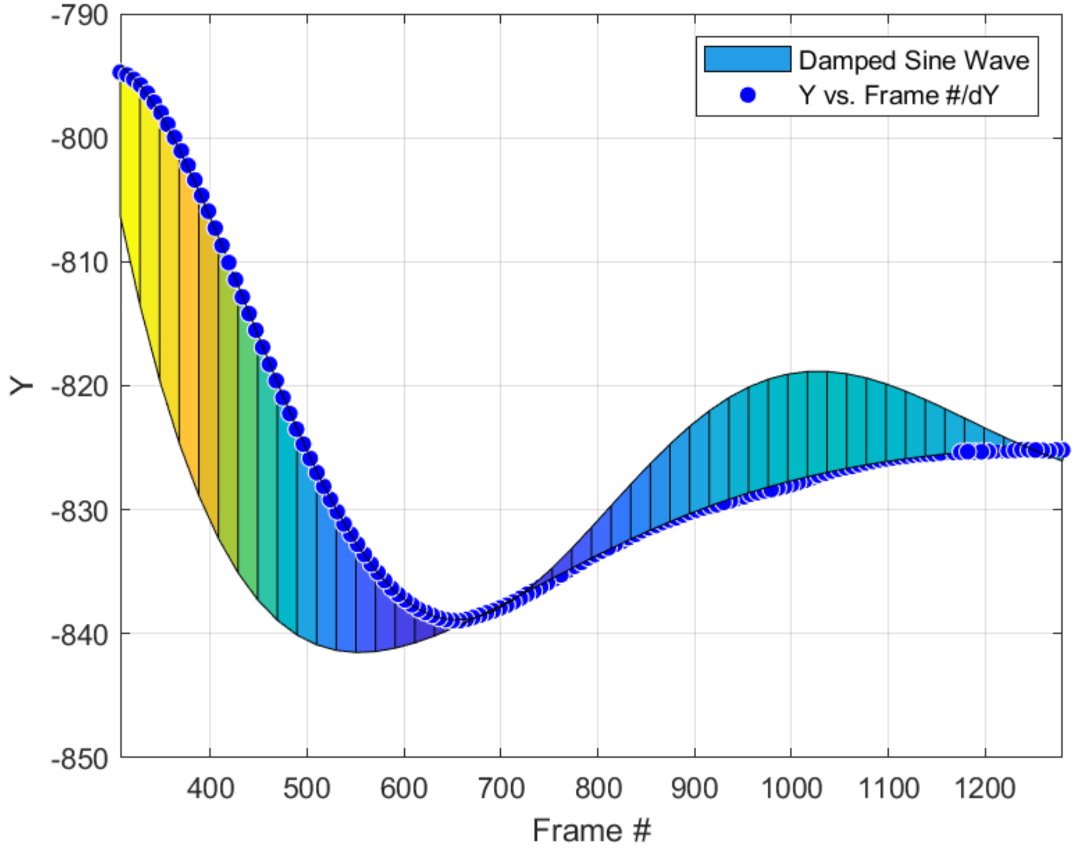


Alternative Fitting Function: Completely Dissociated Compression/Rebound

- **In order to combat fit uniqueness issues, upper and lower bounds for fit coefficients were tightened:**



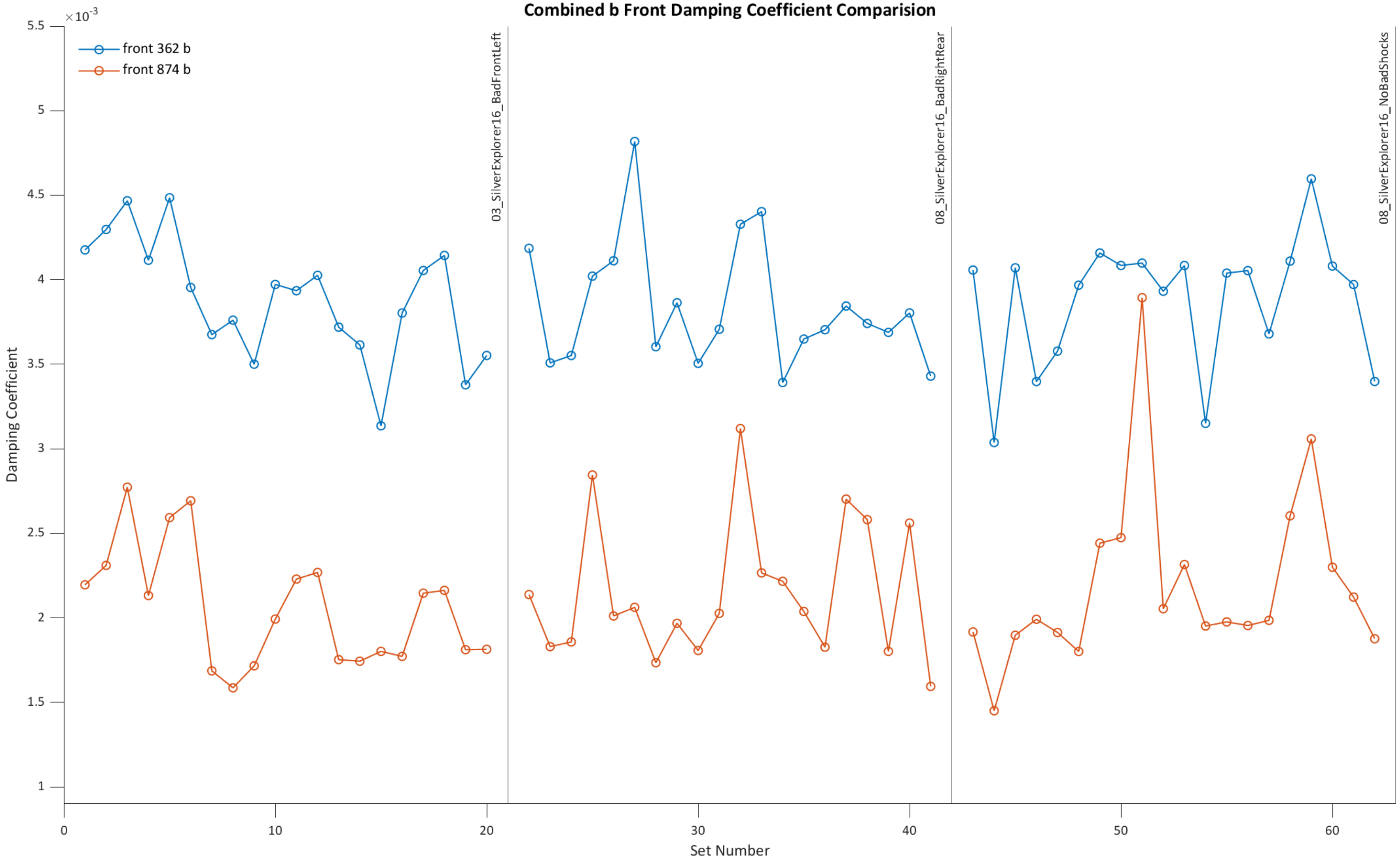
Original Fit Bounds



Tightened Fit Bounds

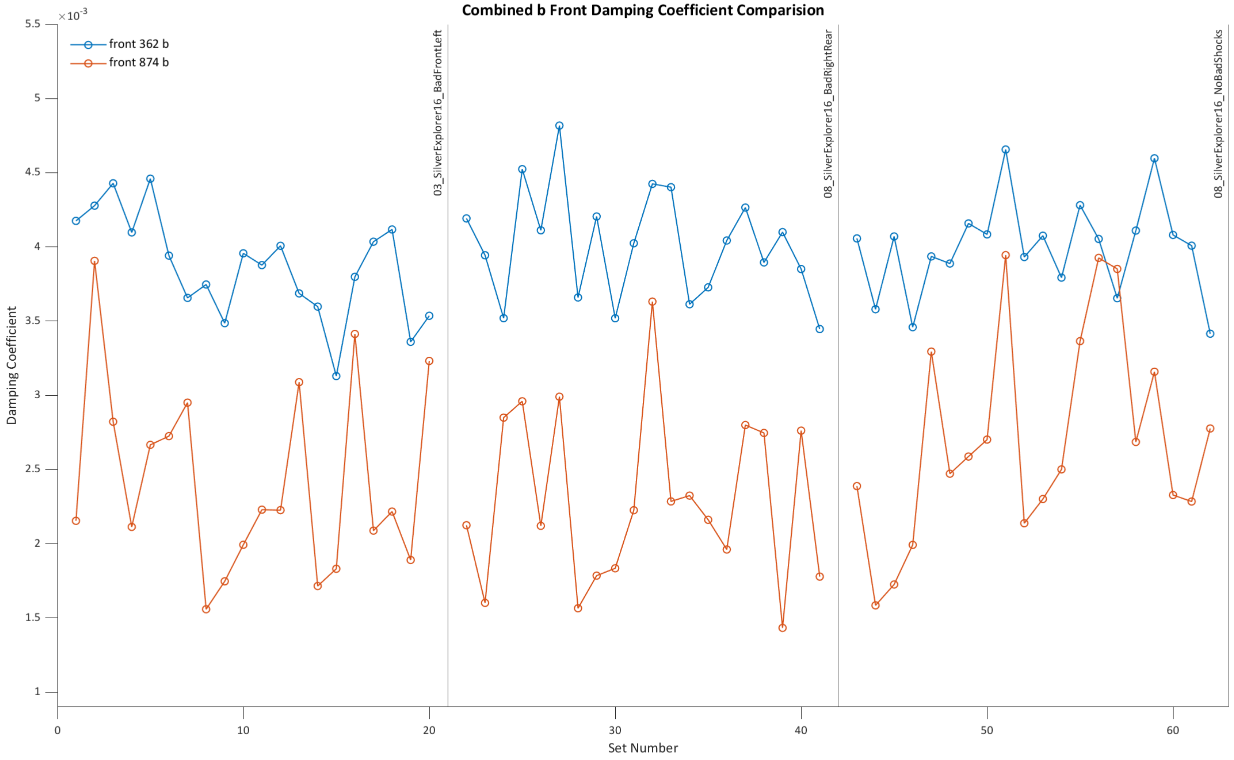
Alternative Fitting Function: Completely Dissociated Compression/Rebound

- **874 results might have improved while 362 results worsened:**

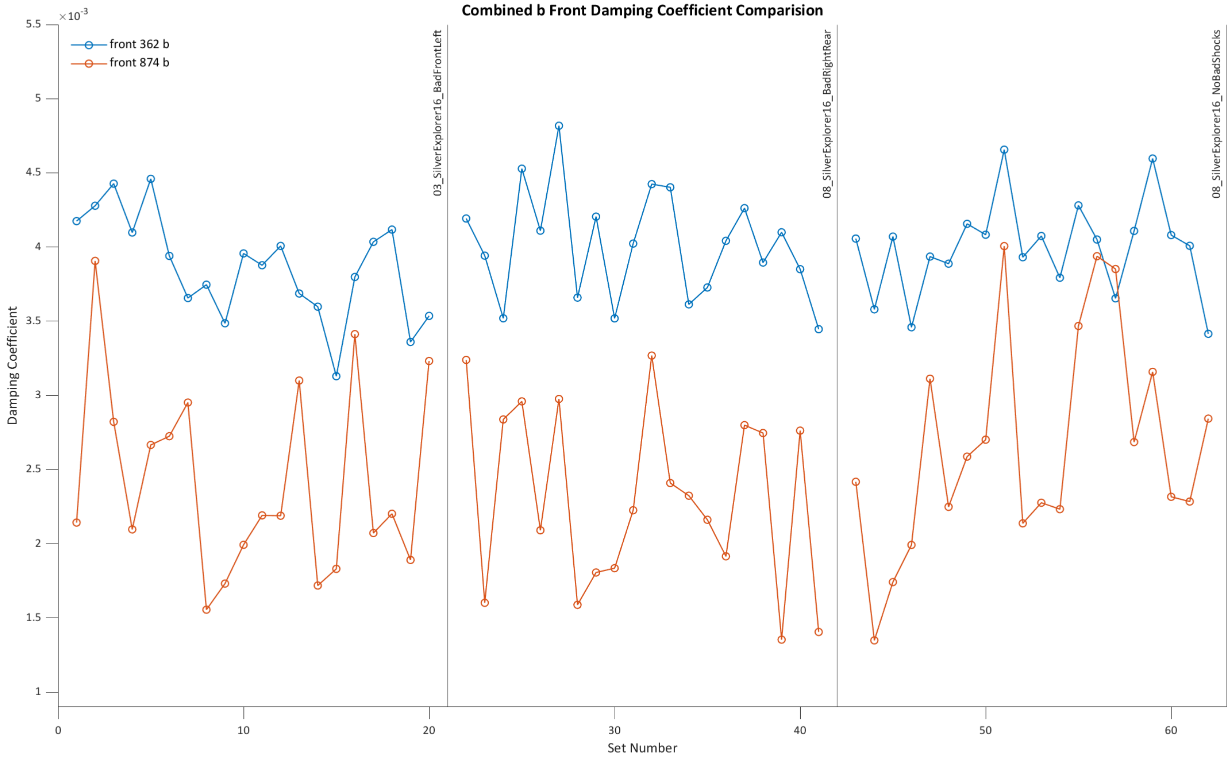


Alternative Fitting Function: Completely Dissociated Compression/Rebound

- **Multiple iterations of various start points and fit bounds didn't seem to improve results**



Iteration 5



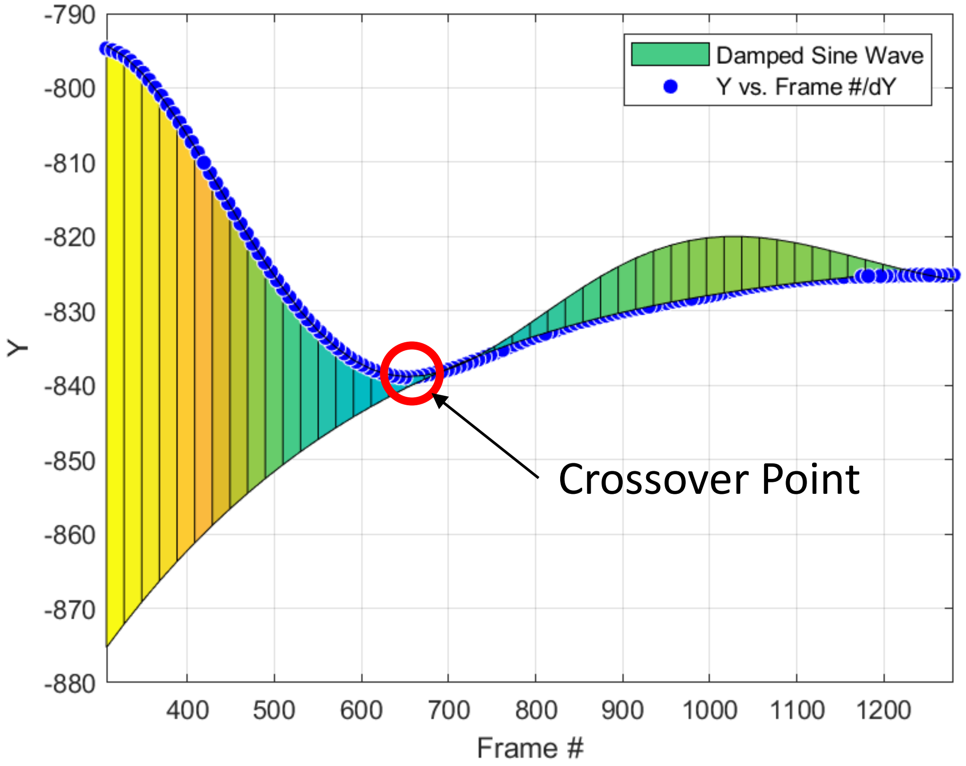
Iteration 6

Fitting Method:
Underdamped Compression,
Critically Damped Rebound

Alternative Fitting Function: Underdamped Compression/Critically Damped Rebound

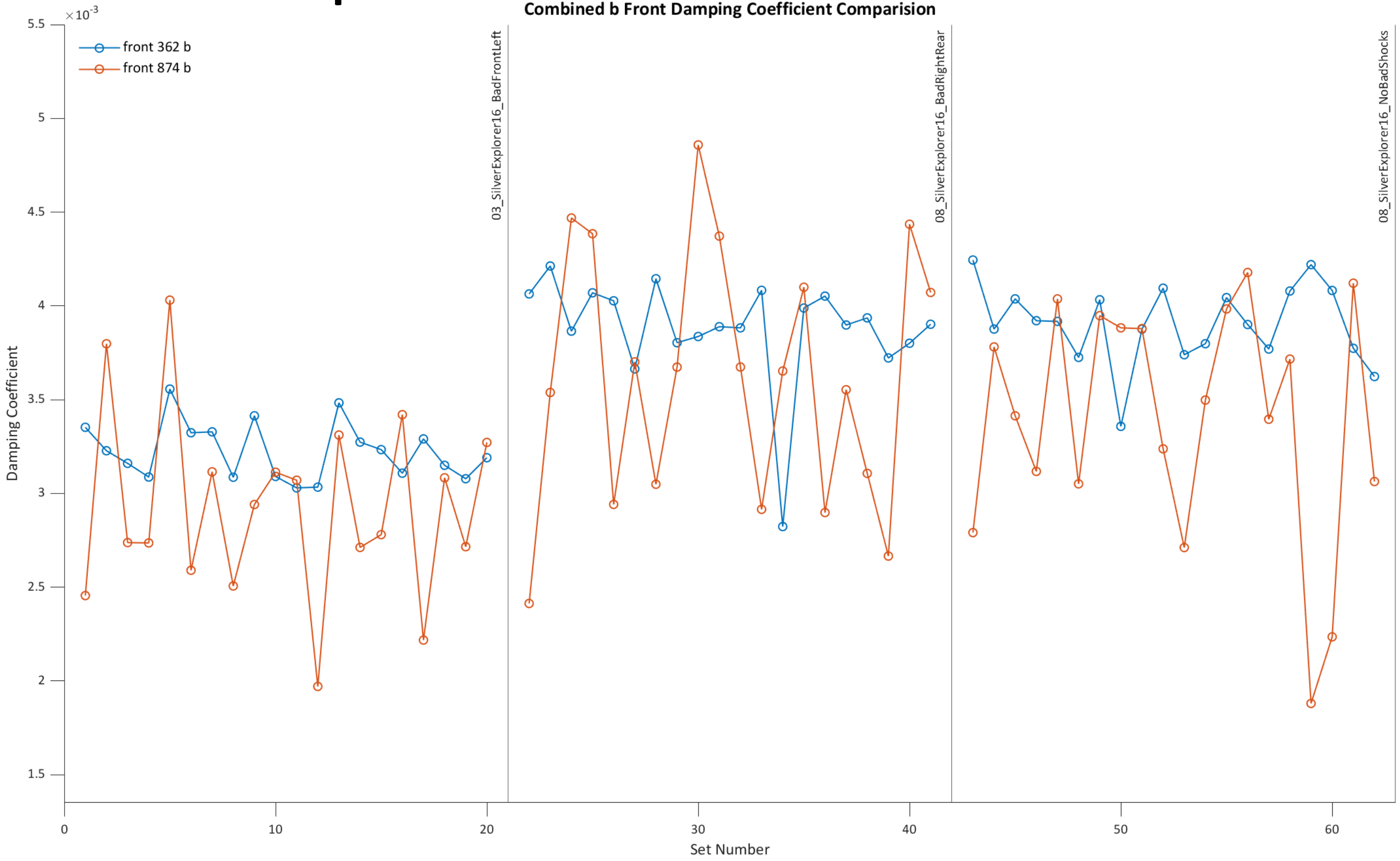
- **Since front shocks didn't have a prominent second hump, tried a fitting function that was critically damped on rebound:**

$$y(t) = \begin{cases} A_a e^{-at} + B_a t e^{-at} & dy \geq 0 \\ A_b e^{-bt} \cos(\omega_b t + \theta_b) & dy < 0 \end{cases}$$

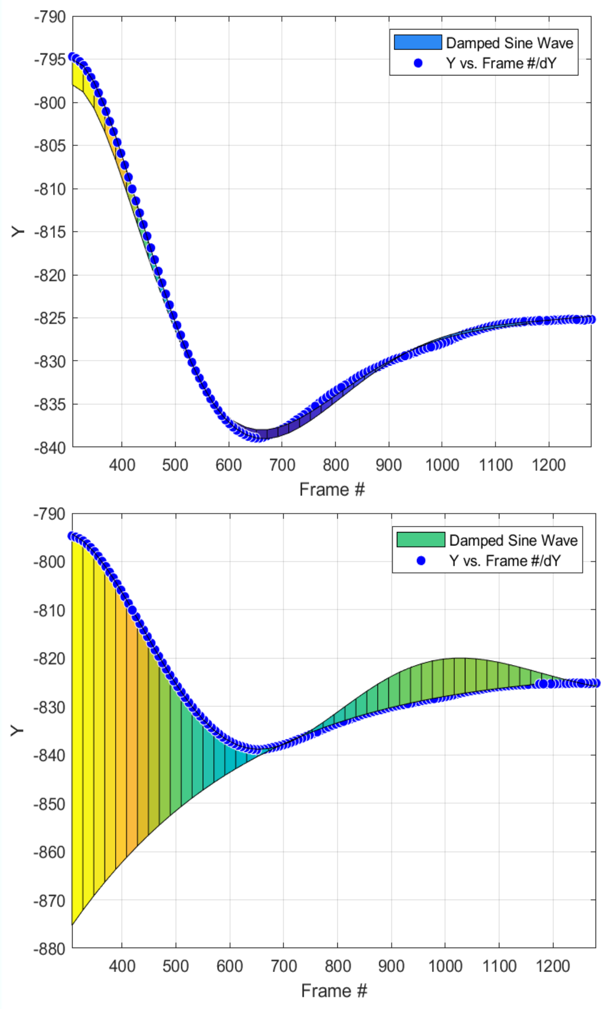


Alternative Fitting Function: Underdamped Compression/Critically Damped Rebound

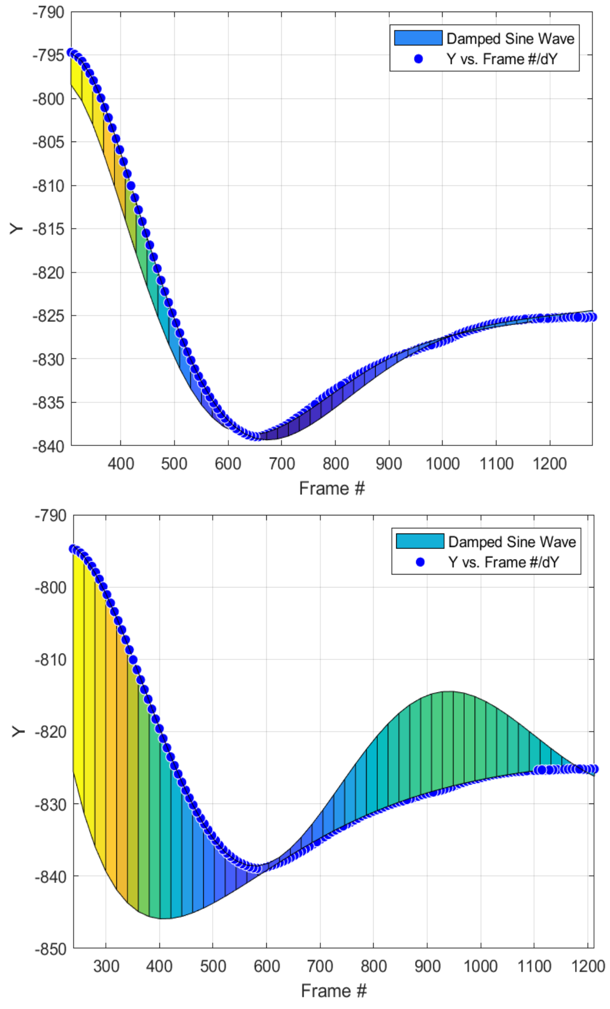
- **Similar issues with fit uniqueness exist**



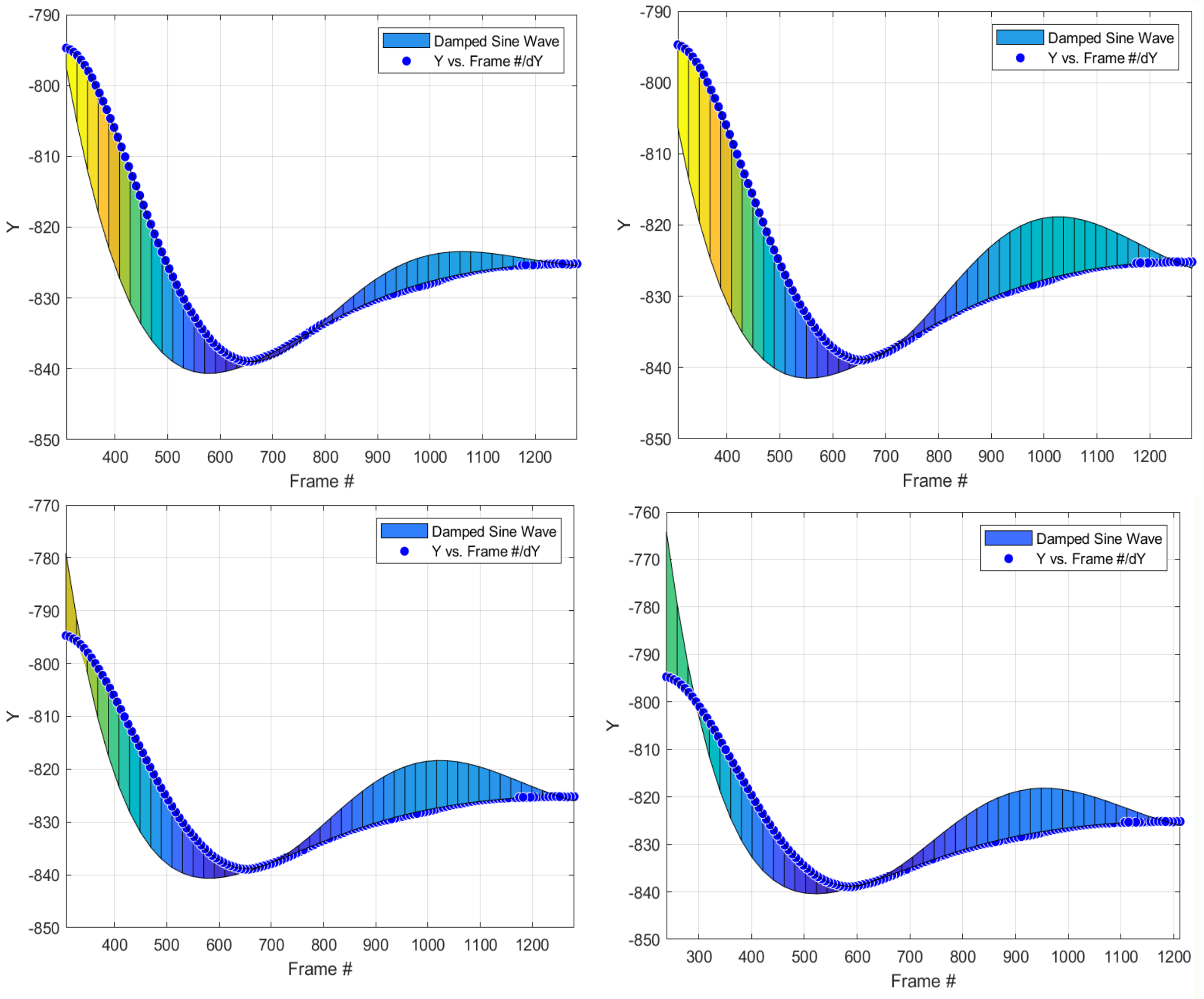
Fit Uniqueness and MATLAB Fitting



Starting Point (above)
Underdamped/Critical (below)



Dual Frequency (above)
Underdamped/Critical (below)



Completely dissociated compression-rebound
(All four)

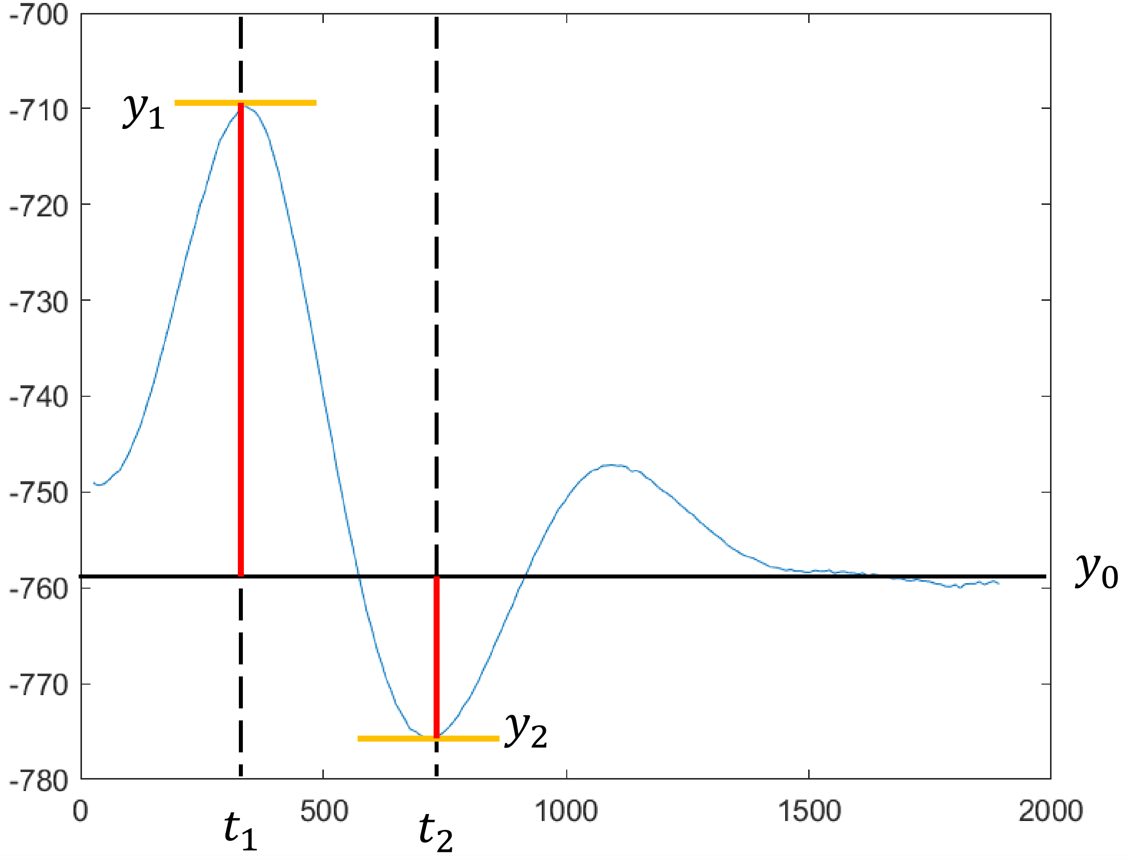
Coefficient Calculation

Fit Uniqueness Possible Solution: Coefficient Calculation

- It may be possible to calculate the four unknowns in our fit equation:

$$y(t) = Ae^{-at}\cos(\omega t) + Be^{-at}\sin(\omega t)$$

- We know four boundary conditions for each half-period in the damped sine:
  - $y(0) = y_1 - y_0$
  - $y(\Delta t) = y_2 - y_0$
  - $\dot{y}(0) = 0$
  - $\dot{y}(\Delta t) = 0$



Fit Uniqueness Possible Solution: Coefficient Calculation

- **System of equations after plugging in boundary conditions:**

$$y_1 - y_0 = A$$

$$y_2 - y_0 = Ae^{-a\Delta t}\cos(\omega\Delta t) + Be^{-a\Delta t}\sin(\omega\Delta t)$$

$$0 = \omega B - aA$$

$$0 = e^{-a\Delta t}[\cos(\omega\Delta t)(\omega B - aA) - \sin(\omega\Delta t)(\omega A + aB)]$$

- **No unique solution exists**
  - Reason behind fit uniqueness issues
- **Highly dependent on calculating ride height correctly**

Dimensionless Fit Parameter

Fit Uniqueness Possible Solution: Dimensionless Fit Parameter

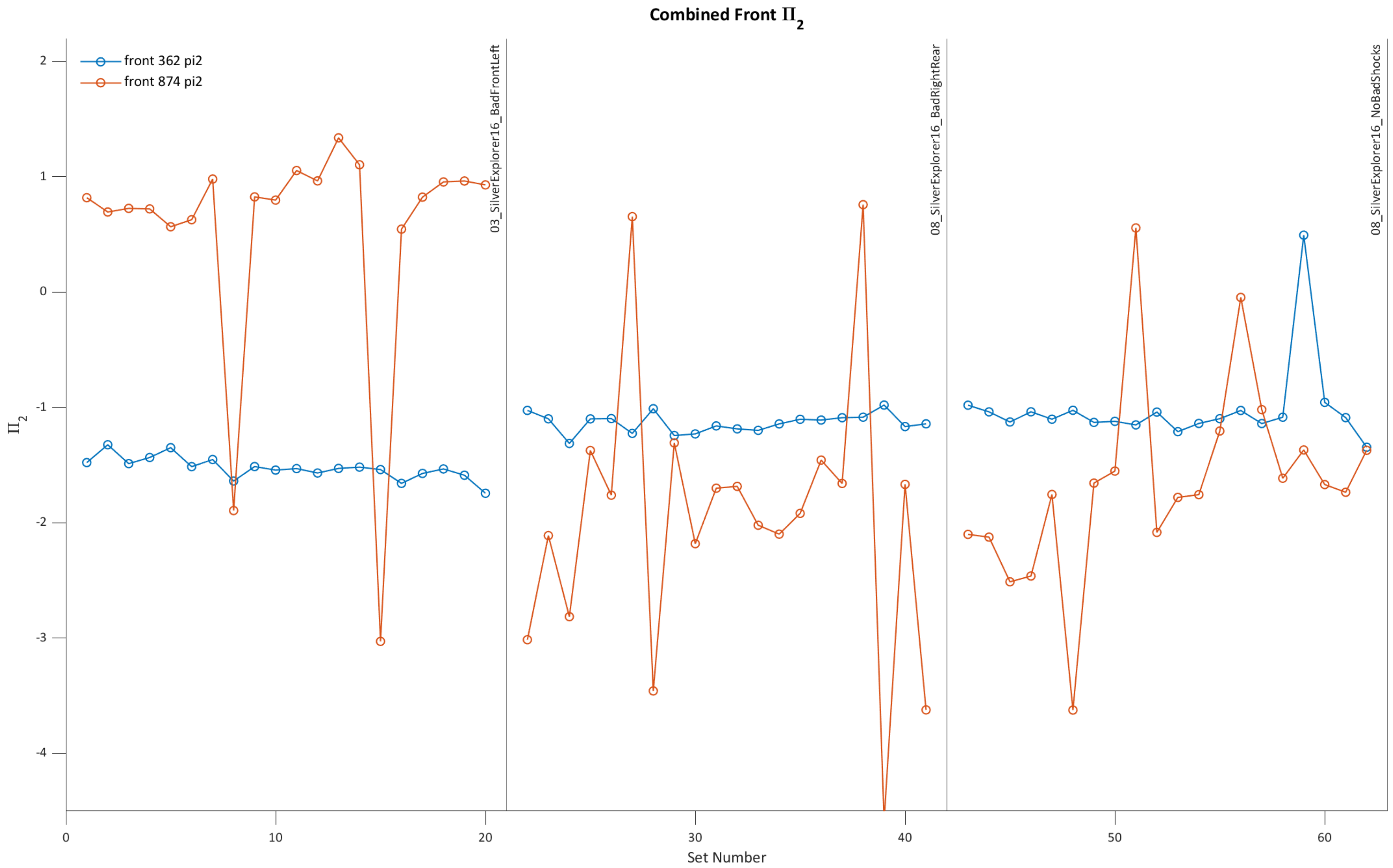
- **Possible method to deal with fit uniqueness issue is to create a new parameter using existing fit coefficients to describe the damping state of the vehicle track**
- **Fit equation:**

$$y(t) = \begin{cases} A_a e^{-at} \cos(\omega_a t + \theta_a) & dy \geq 0 \\ A_b e^{-bt} \cos(\omega_b t + \theta_b) & dy < 0 \end{cases}$$

- **Fit coefficients:**
  - A – Amplitude [1]
  - a – Rebound damping coefficient [1/s]
  - b – Compression damping coefficient [1/s]
  - ω – Damping frequency [rad/s]
  - θ – Phase angle [rad]
- $\Pi_1 = \frac{\omega_a}{a\theta_a}$ $\Pi_4 = A_b$ $\Pi_7 = \frac{A_b\omega_b}{b\theta_b}$
- $\Pi_2 = \frac{\omega_b}{b\theta_b}$ $\Pi_5 = \frac{a}{b}$ $\Pi_8 = \frac{\omega_a}{\omega_b}$
- $\Pi_3 = A_a$ $\Pi_6 = \frac{A_a\omega_a}{a\theta_a}$ $\Pi_9 = \frac{\theta_a}{\theta_b}$

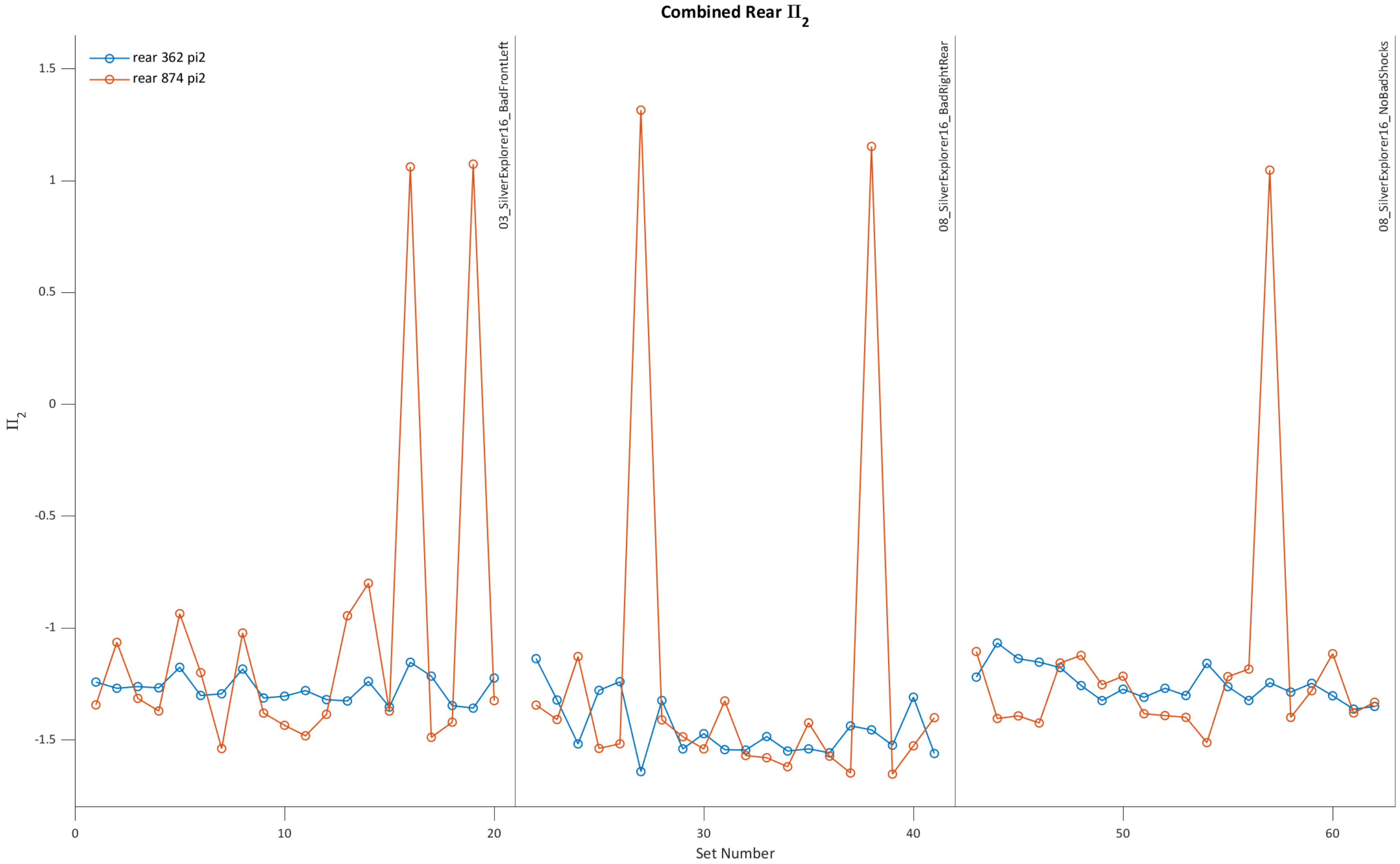
Fit Uniqueness Possible Solution: Dimensionless Fit Parameter

- **Front left Π_2 shows promise in classification use, front right is unusable**

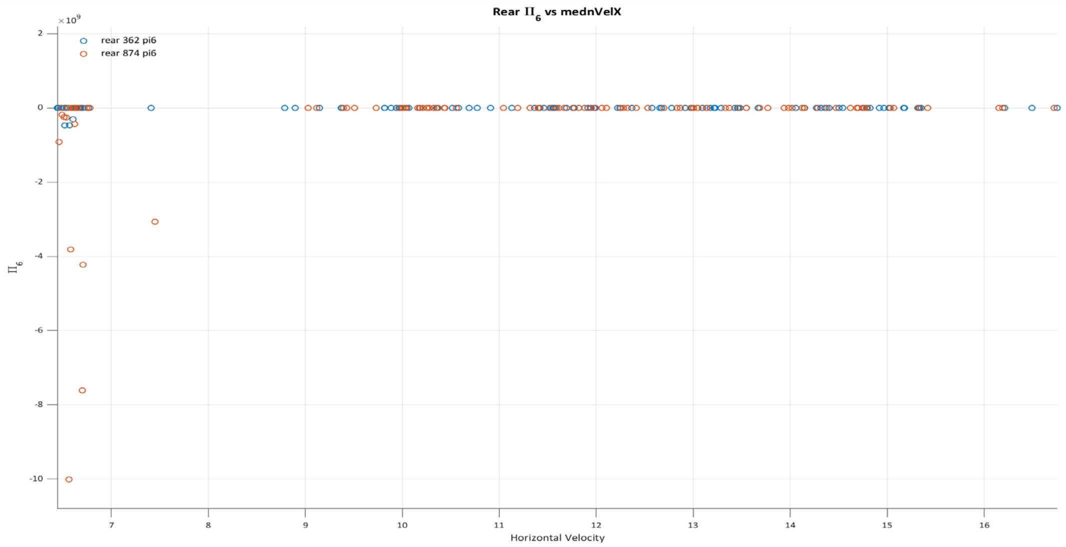


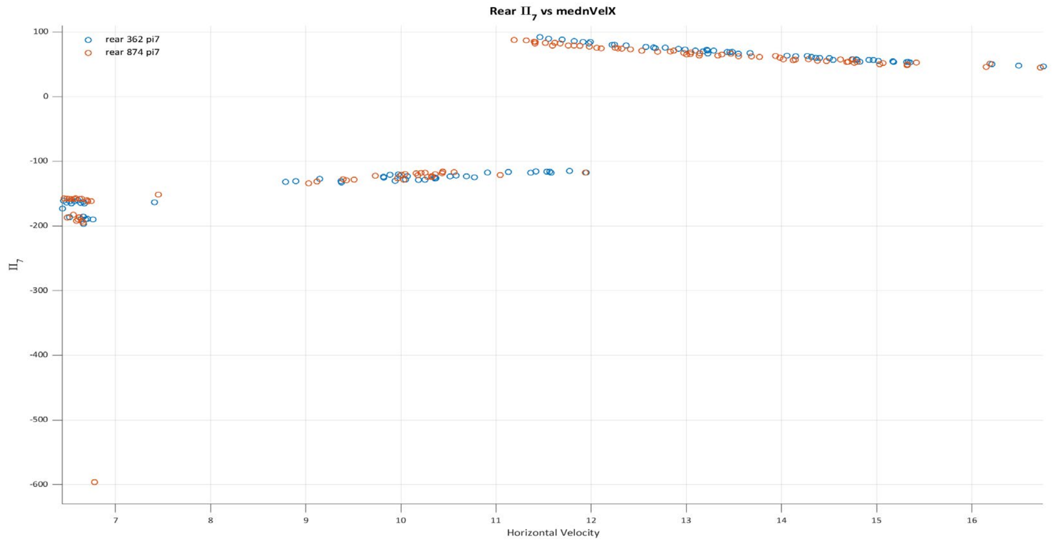
Fit Uniqueness Possible Solution: Dimensionless Fit Parameter

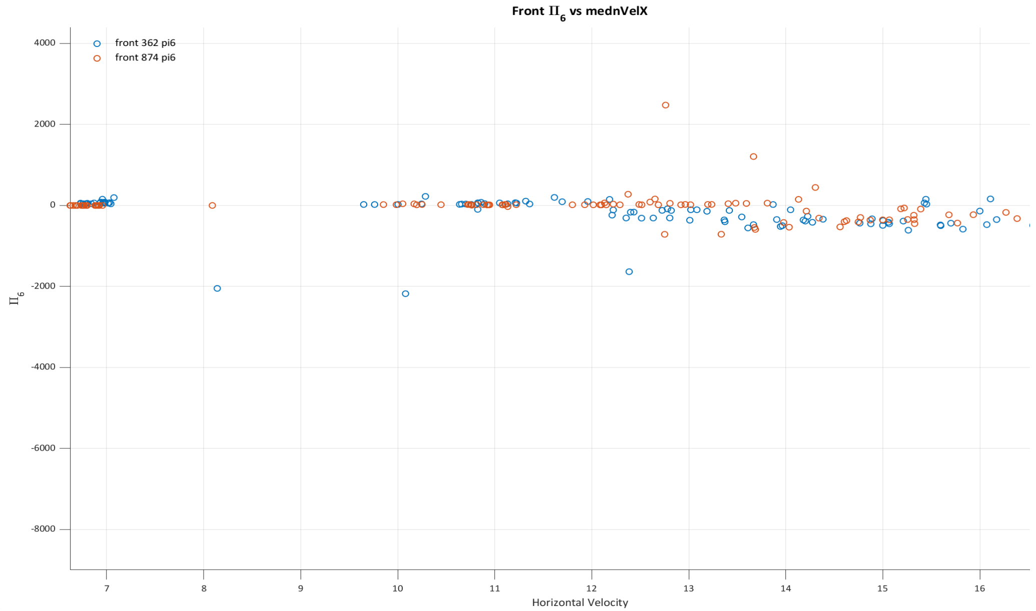
- **Rear constants are not as consistent as front left**

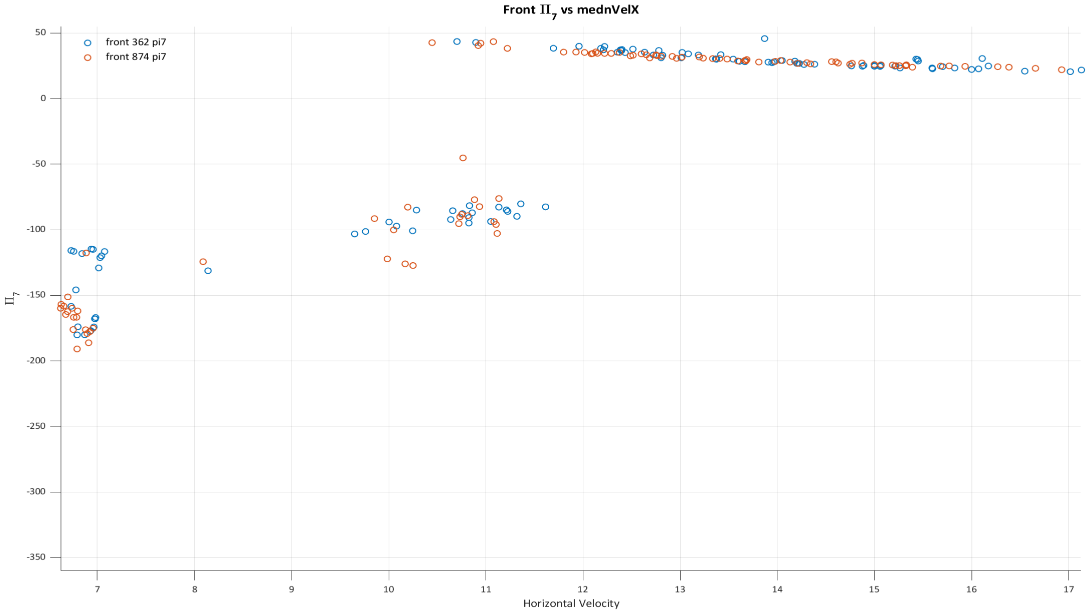


Fit Uniqueness Possible Solution: Dimensionless Fit Parameter



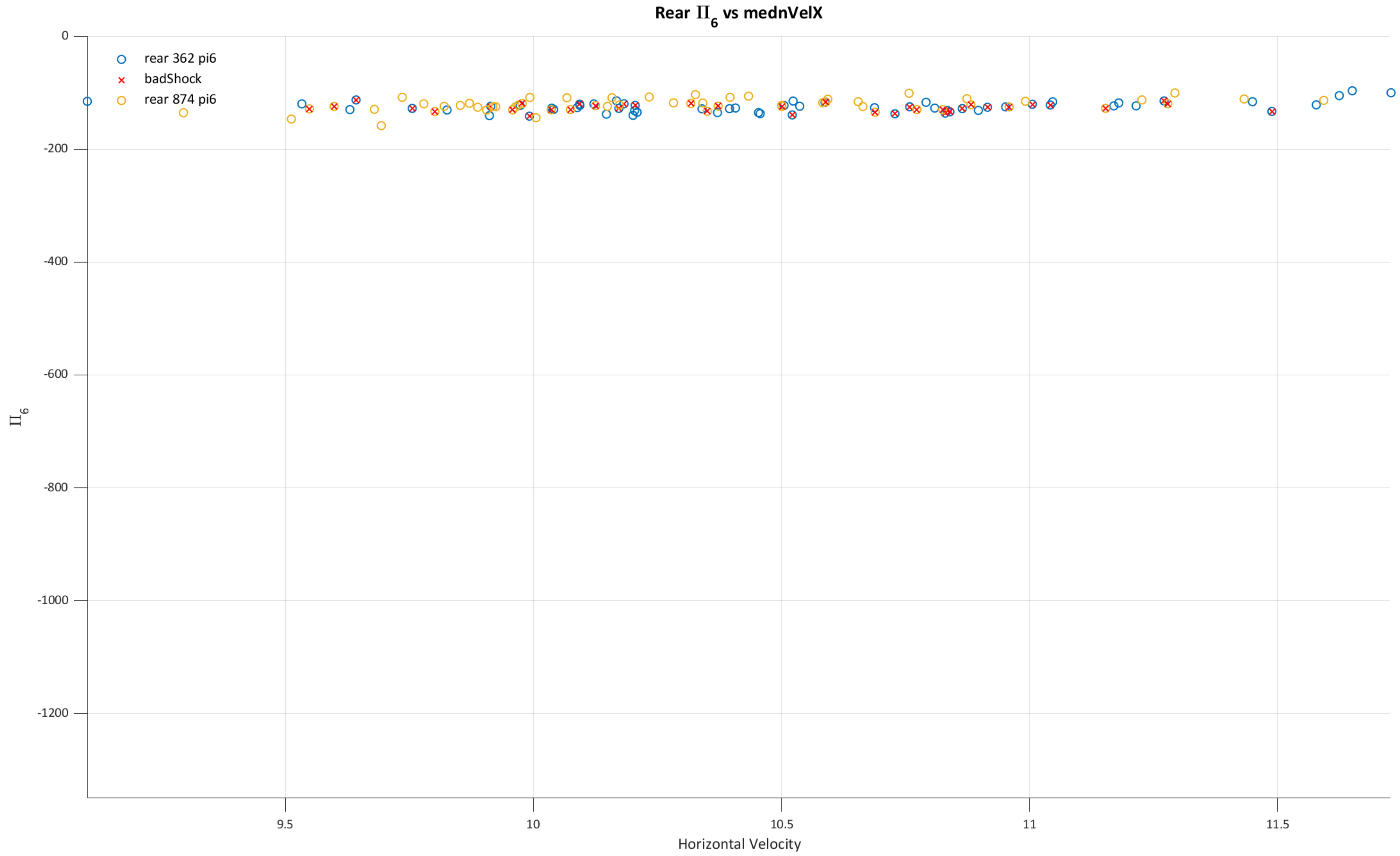






Fit Uniqueness Possible Solution: Dimensionless Fit Parameter

- **Bad shocks mix with good shocks when looking at horizontal velocity correlation**



Timeout

- **Fitting related analysis methods seem to have one issue or another for the front shocks**
  - MATLAB fitting quirks (tightening bounds, changing start points)
  - Fit Uniqueness
  - No unique solutions
  - Unclear dimensionless parameter relationships
- **"What was unique in all of the fits that were attempted?"**
  - The track of the vehicle body
- **"What do dampers do?"**
  - Dissipate energy at a controlled rate
- **"What is a characteristic of bad shocks?"**
  - They are MORE BOUNCY

Energy Dissipation Calculation

Energy Dissipation Method

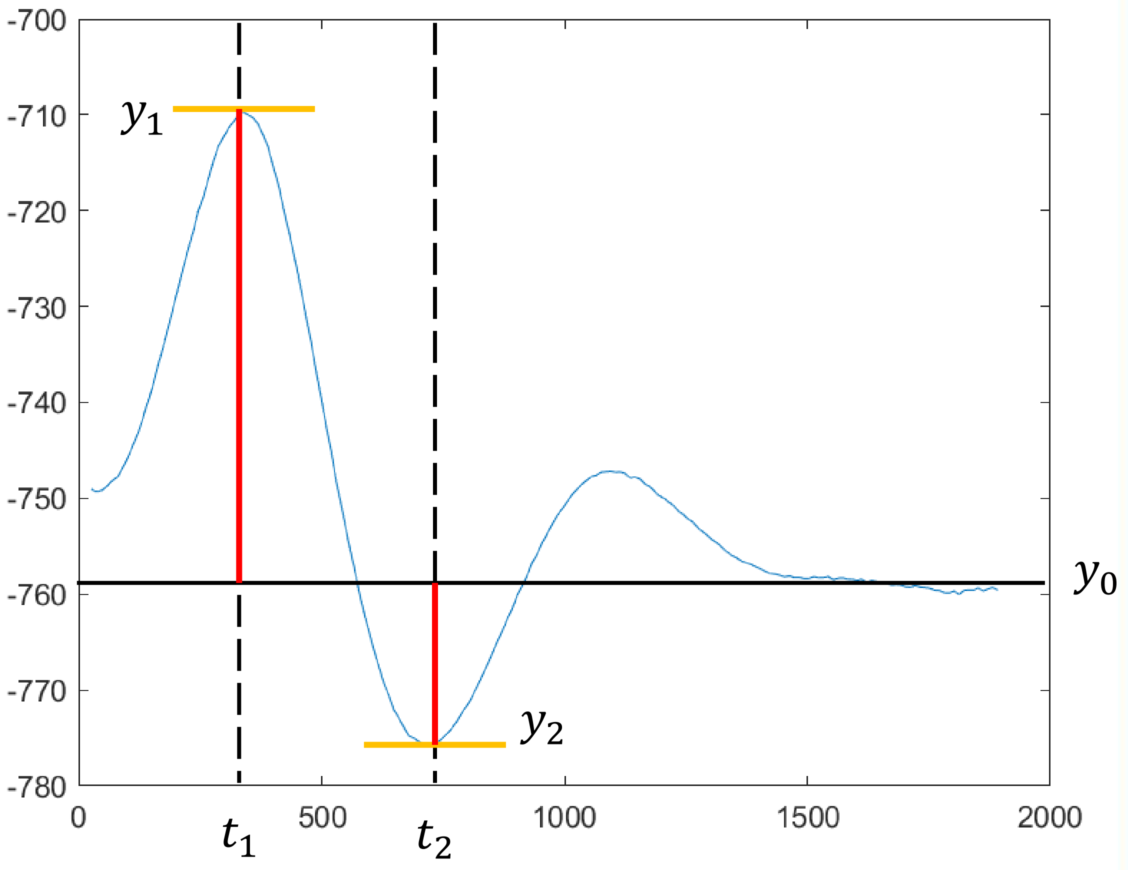
- **Energy stored in a spring:**

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k(y_1 - y_0)^2$$

- **May be able to classify the shocks based on what percentage of energy is dissipated**
- **Compression Energy Dissipation:**

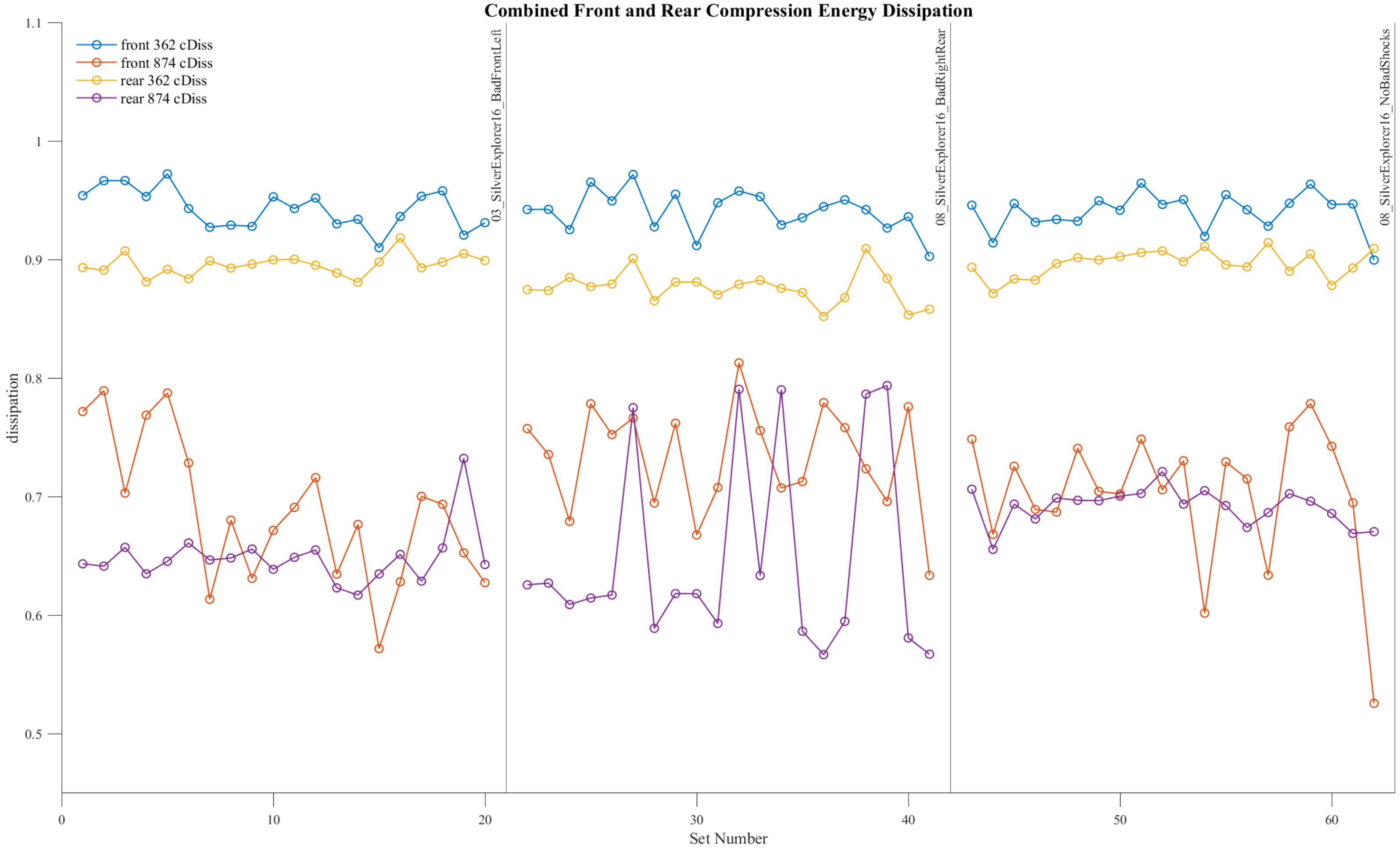
$$E_{dissipated} = \frac{(y_2 - y_0)^2}{(y_1 - y_0)^2}$$

- **Pro:**
  - Relies on data from track itself
- **Con:**
  - Dependent on calculating ride height correctly



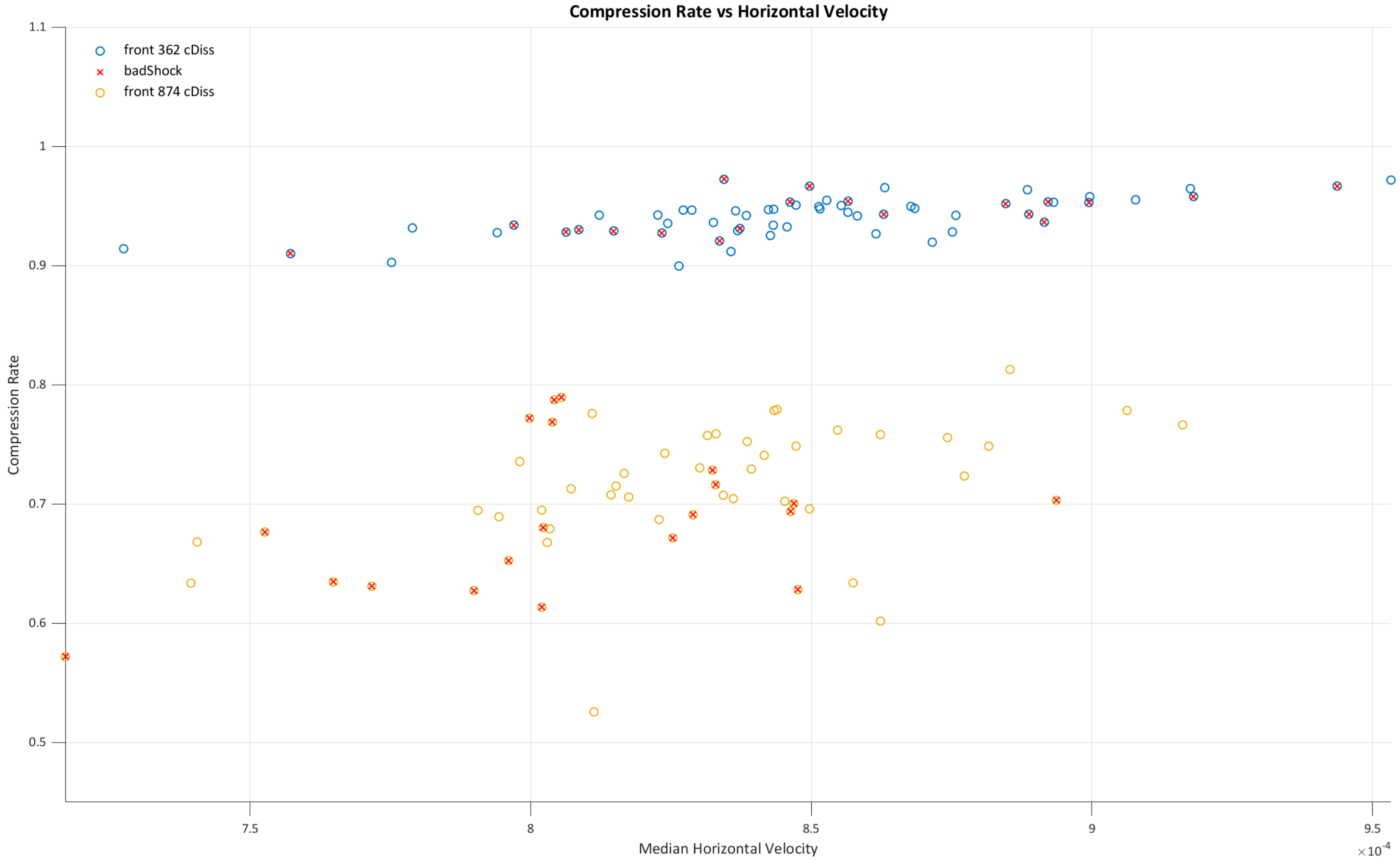
Energy Dissipation Method: Results

- **Due to issues with selecting ride height properly and intrinsic variance, bad shock cases are not clearly definable**



Energy Dissipation Method: Results

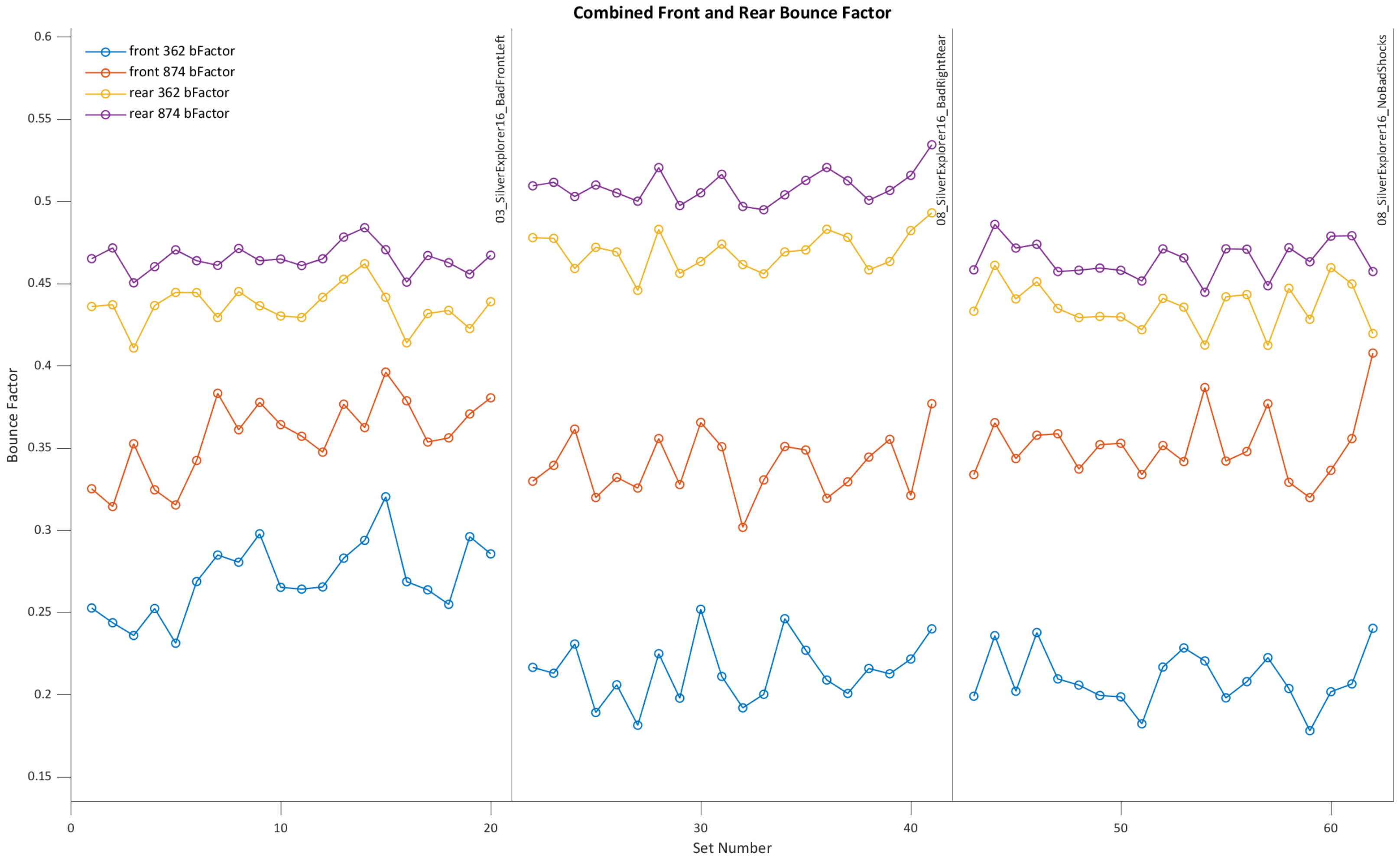
- **Compression energy dissipation doesn't appear to rely on velocity**



Bounciness Method

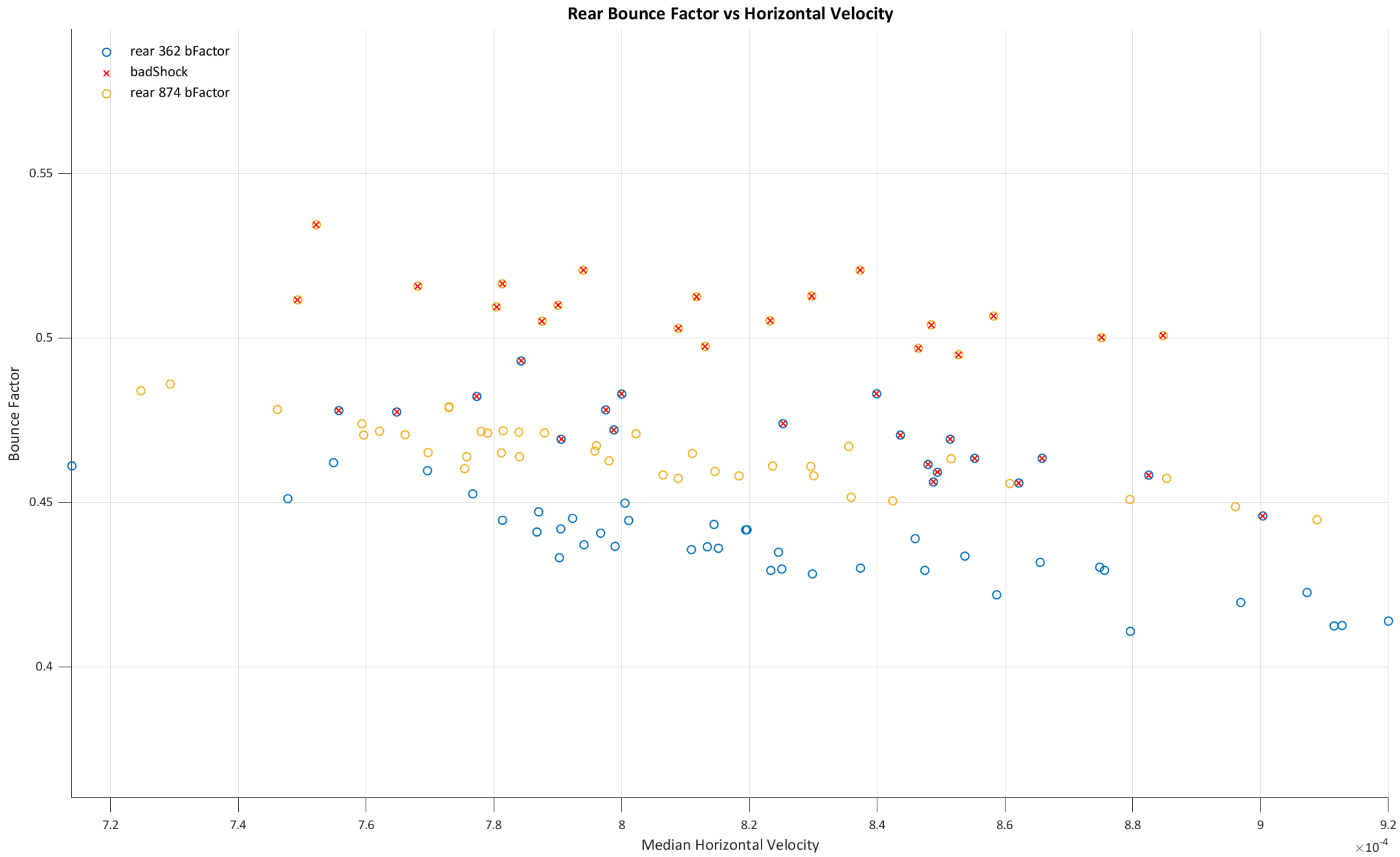
Bounce Classification Method

- It may be possible to calculate a "Bounce Factor (B)" from track data
- First attempt was the ratio of the rebound distance to the compression distance
- Bad shock cases have a higher bounce factor as expected
- Lots of variance within runs
- Still dependent on velocity
- Other investigation avenues
  - Integration of "bounce time" into bounce factor



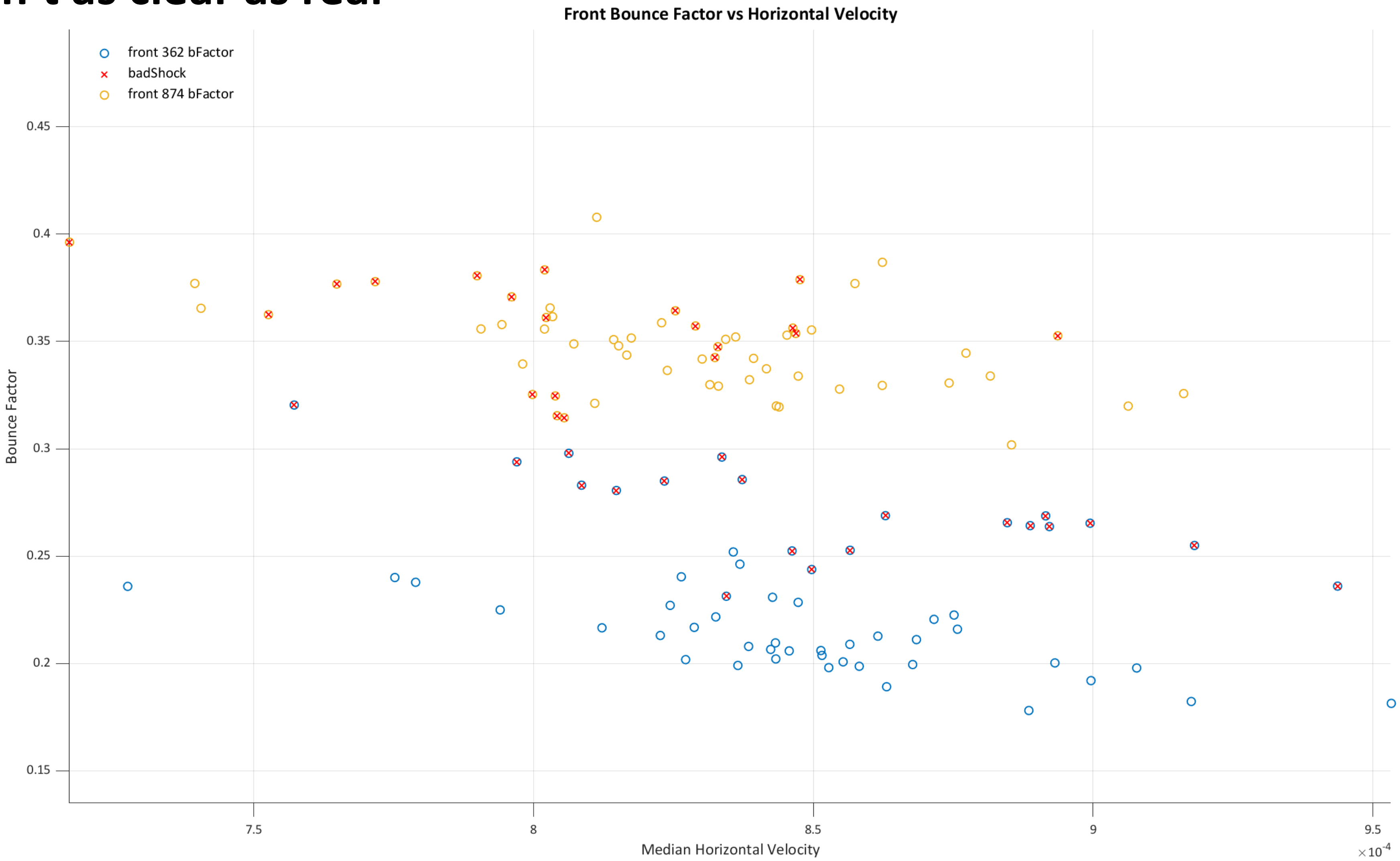
Bounce Classification Method

- **Method similar to damping acceleration could be used to classify rear shock cases**



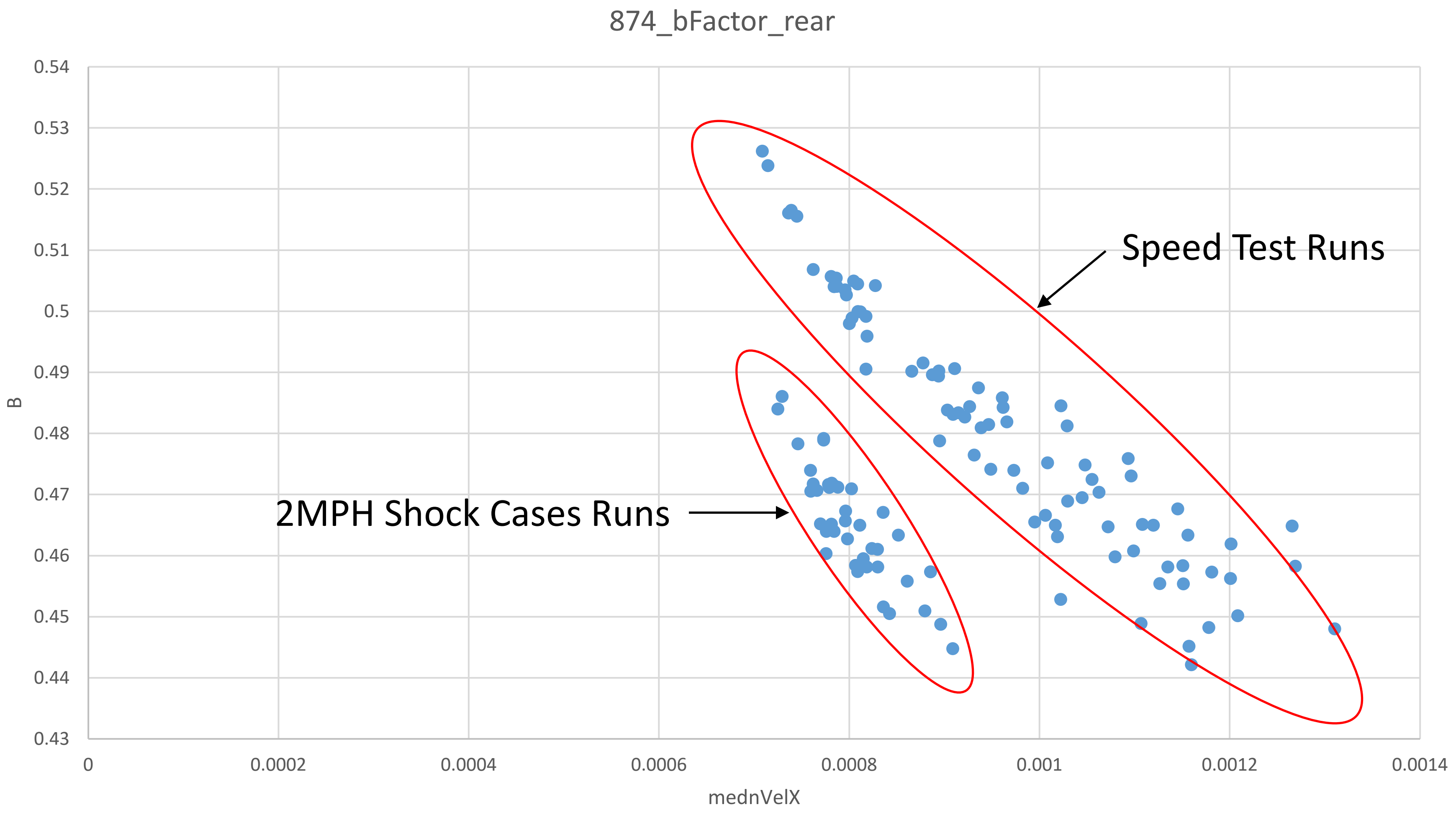
Bounce Classification Method

- **Fronts aren't as clear as rear**



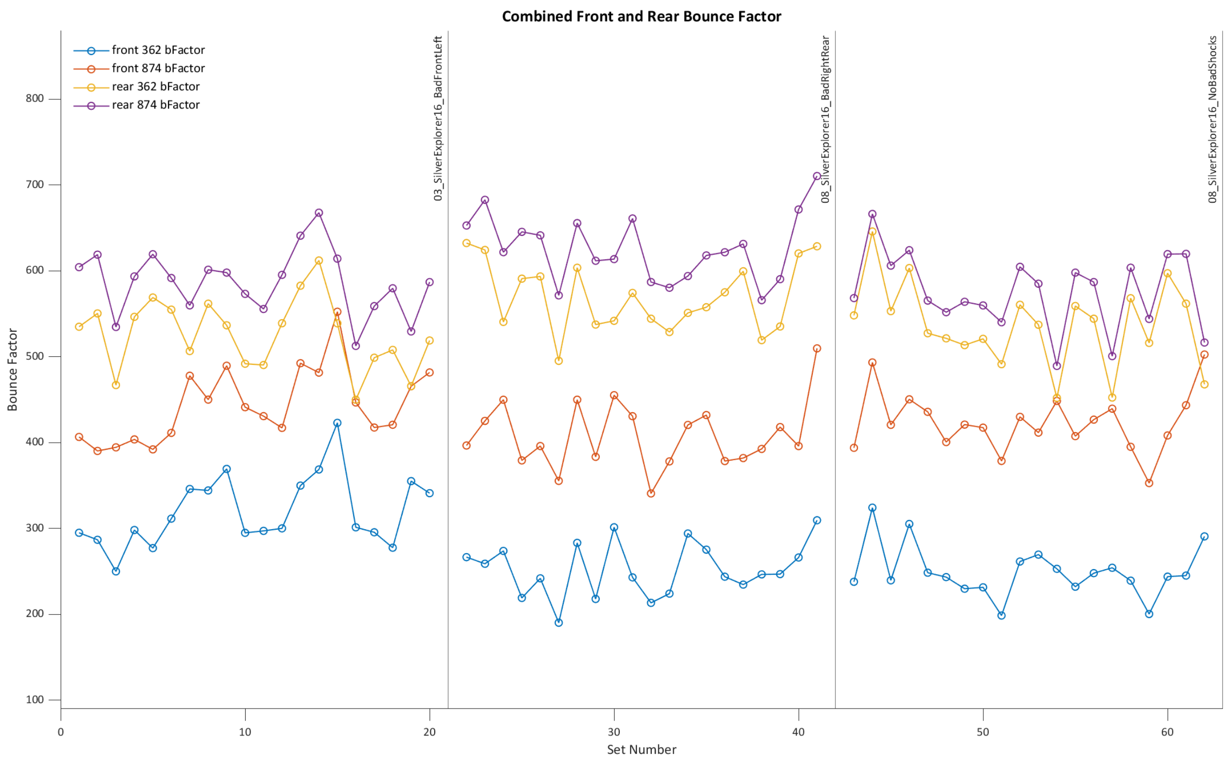
Bounce Classification Method

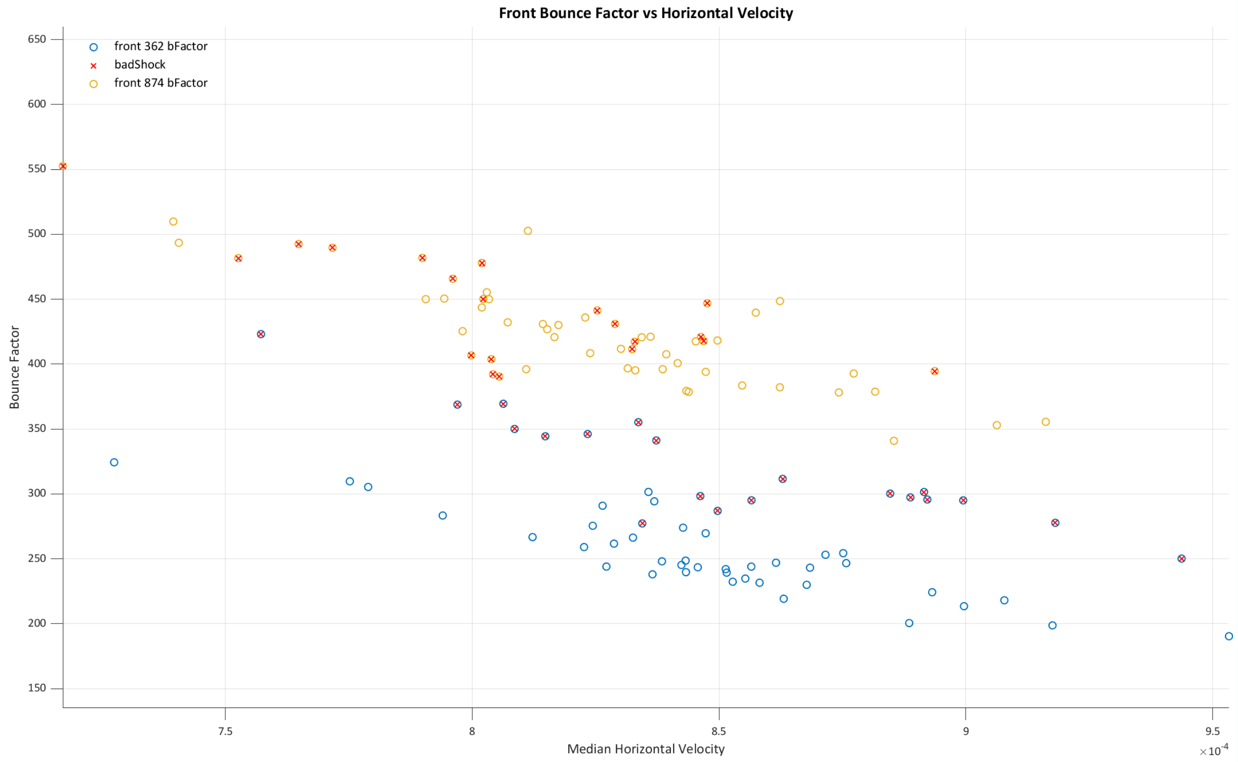
- **Velocity dependence is being impacted by some external factor**



Bounce Classification Method

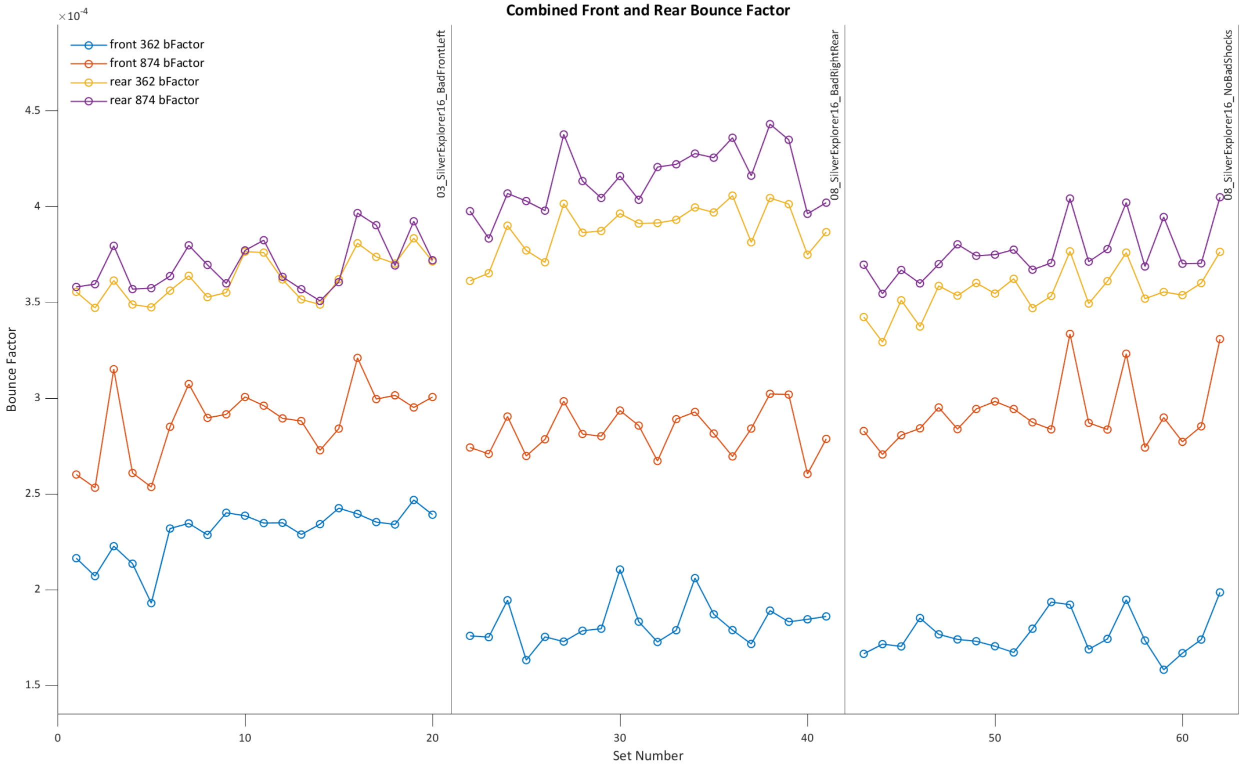
- **Attempts to integrate horizontal velocity into bounce factor did not produce meaningful results.**

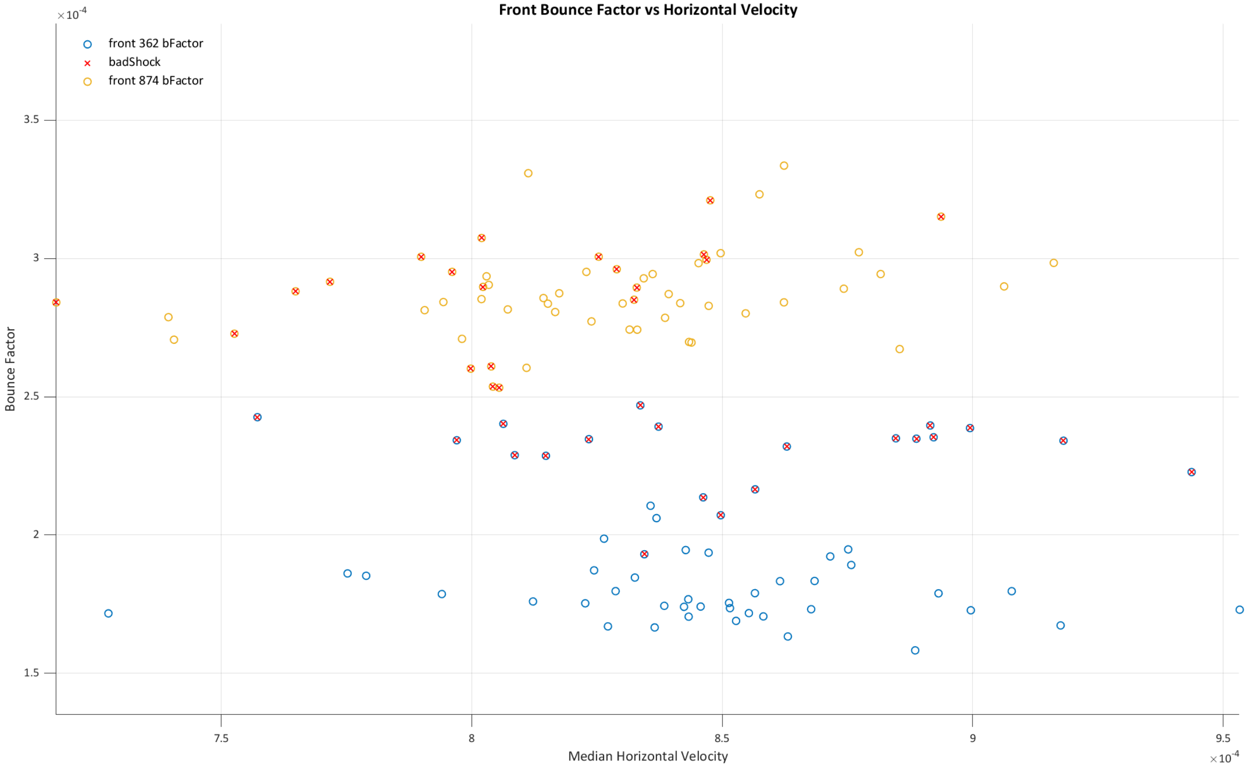




Bounce Classification Method

- **Attempts to integrate horizontal velocity into bounce factor did not produce meaningful results.**





Conclusions

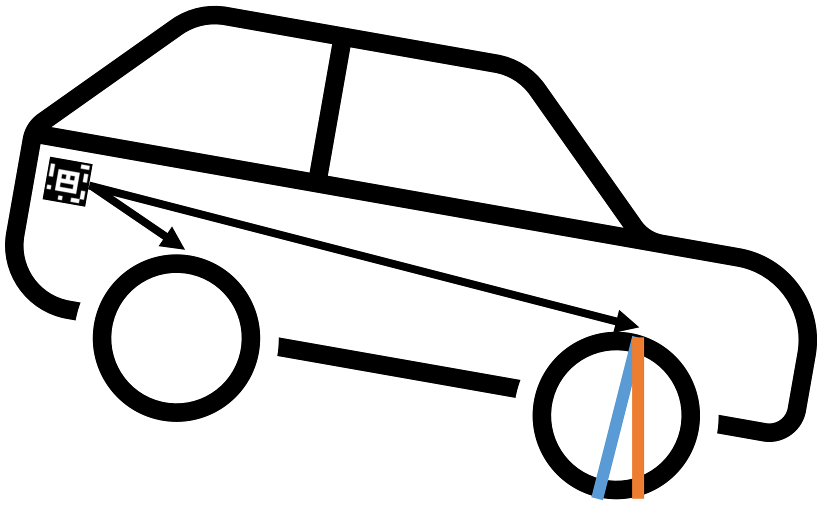
- **Current analysis, correction, and classification methodology is sufficient for diagnosis of rear shocks**
- **The damping acceleration classification method may be used as a reliable alternative to the current velocity correction for the rears, and a less reliable classification method for the front left shock**
- **Upon further testing, the median rebound velocity classification method could prove to be extremely reliable and robust at diagnosing bad front shock cases**
- **Fitting related analysis methods perform with mixed results but in general don’t perform well on front shock cases**
    - Alternative fits
    - Alternative bounds and start points
    - Coefficient calculation
    - Dimensionless parameters

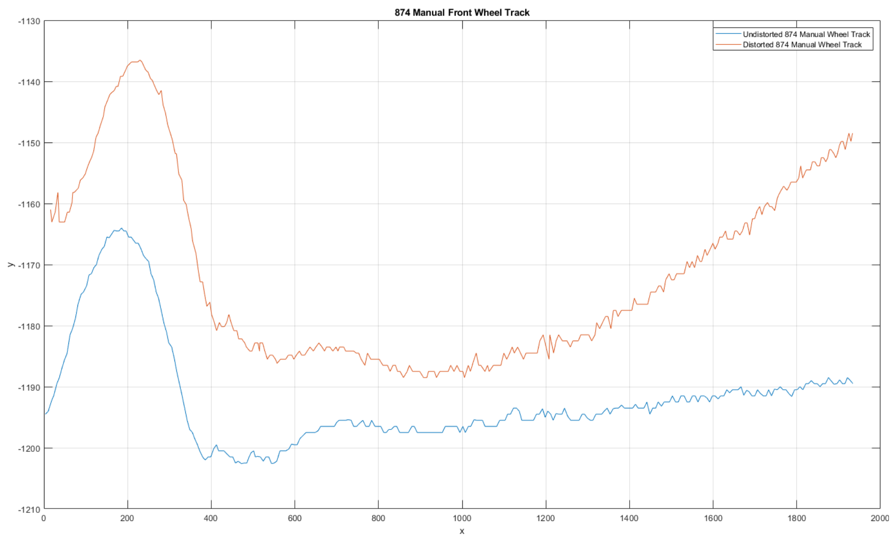
Conclusions

- **Energy Dissipation Method**
  - Need more reliable ride height calculation method
- **Bounce Factor**
  - Shows promise
  - Could be a robust alternative to fitting and damping coefficients
  - Relies on track itself
  - Lots of variation as of now, methods to mitigate that could be investigated
  - External factor affecting velocity correlation needs to be identified

Future Work

- **Potential improvements dependent on AprilTag pose data viability**
  - Can use rigid body kinematics to track any point on the vehicle with potentially one AprilTag or extended front/rear paths based on rear/front AprilTags
  - Can log the rotation rate of the body to fit to more accurate half-car vehicle models
  - Can use the pose data to remove body angle from track
- **Tire compression investigation**
  - Relevance & Effects
- **Bump curvature investigation**
  - Track peak location relative to bump
- **Fitting to a discrete quasi-steady state simulation**
- **Investigation of combination of front/rear tracks for half-car vehicle modeling**

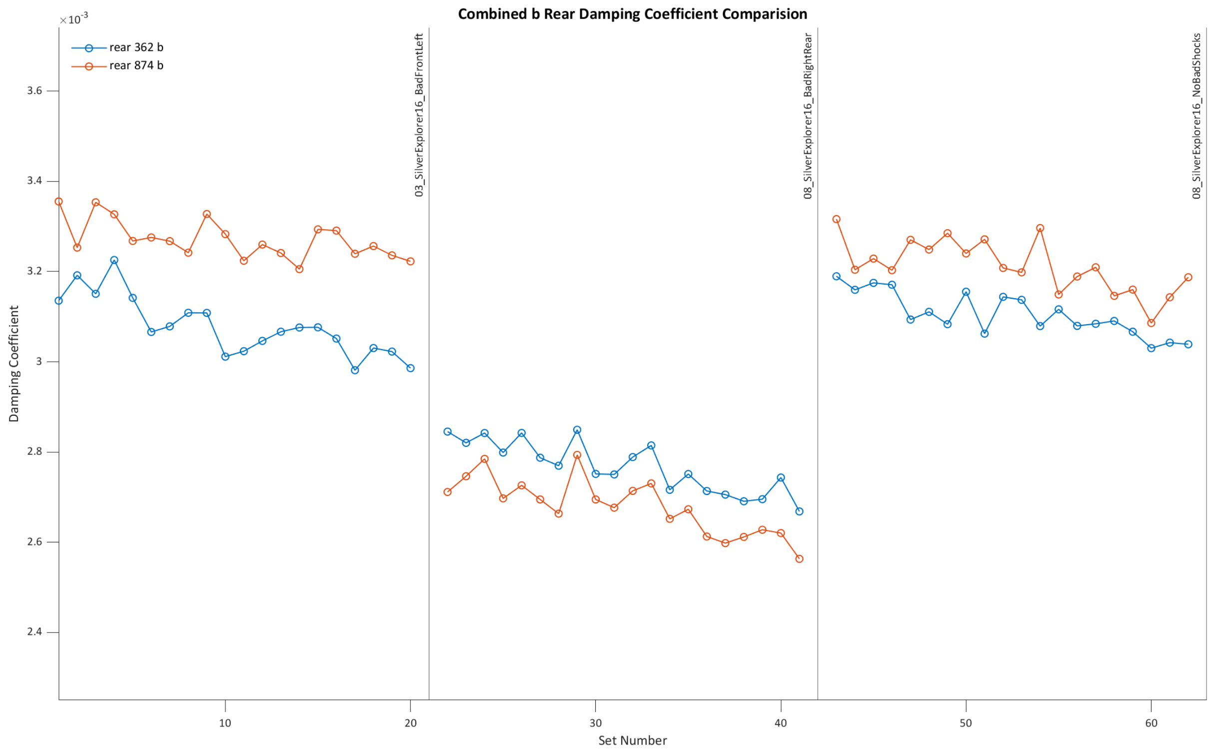




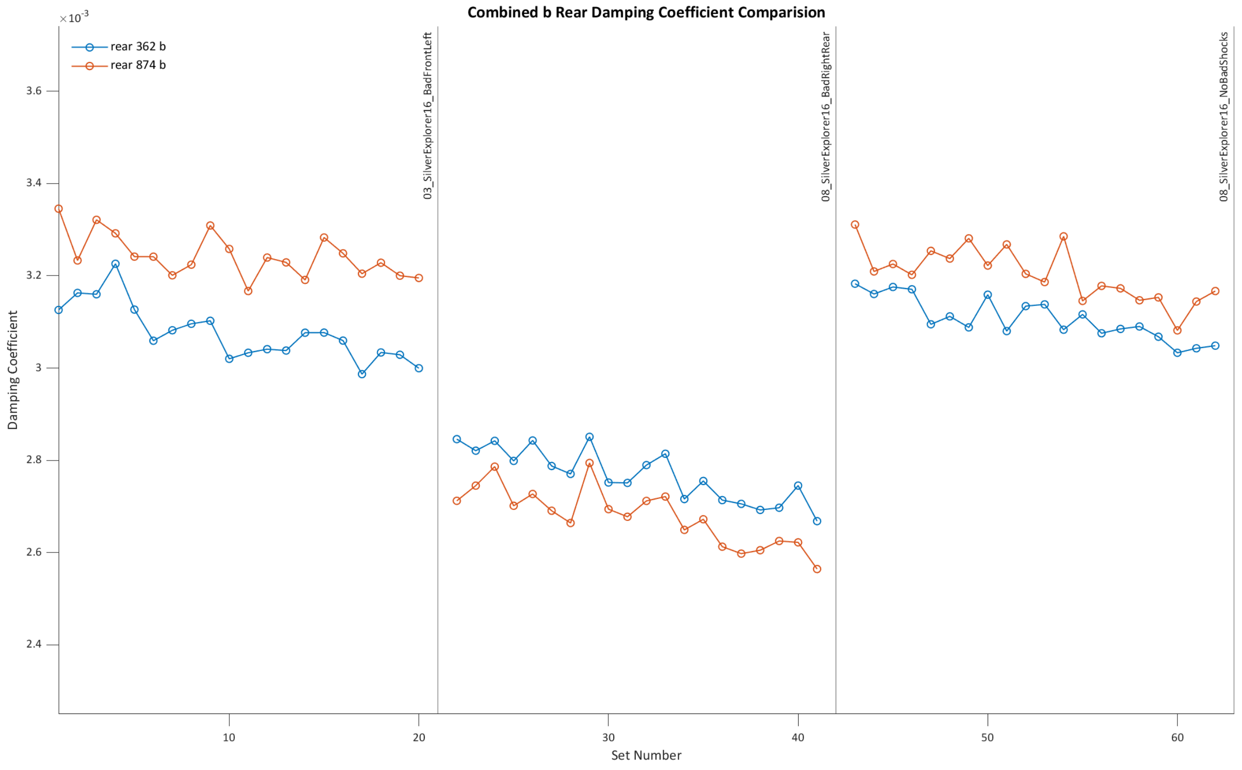
Extra Slides

Alternative Fitting Function: Dual Frequency

- **Very little change in rears due to stronger symmetry**



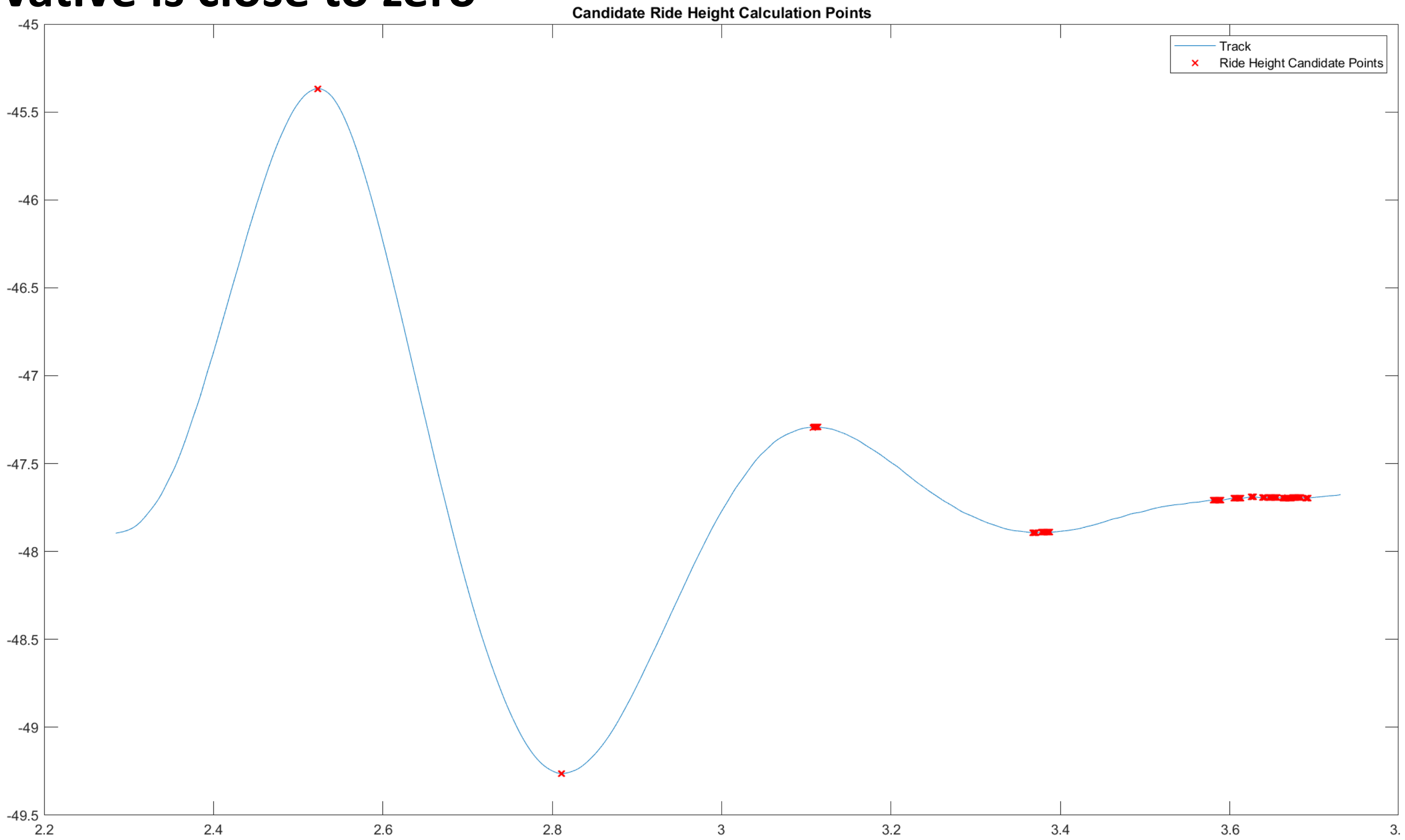
Single Frequency



Dual Frequency

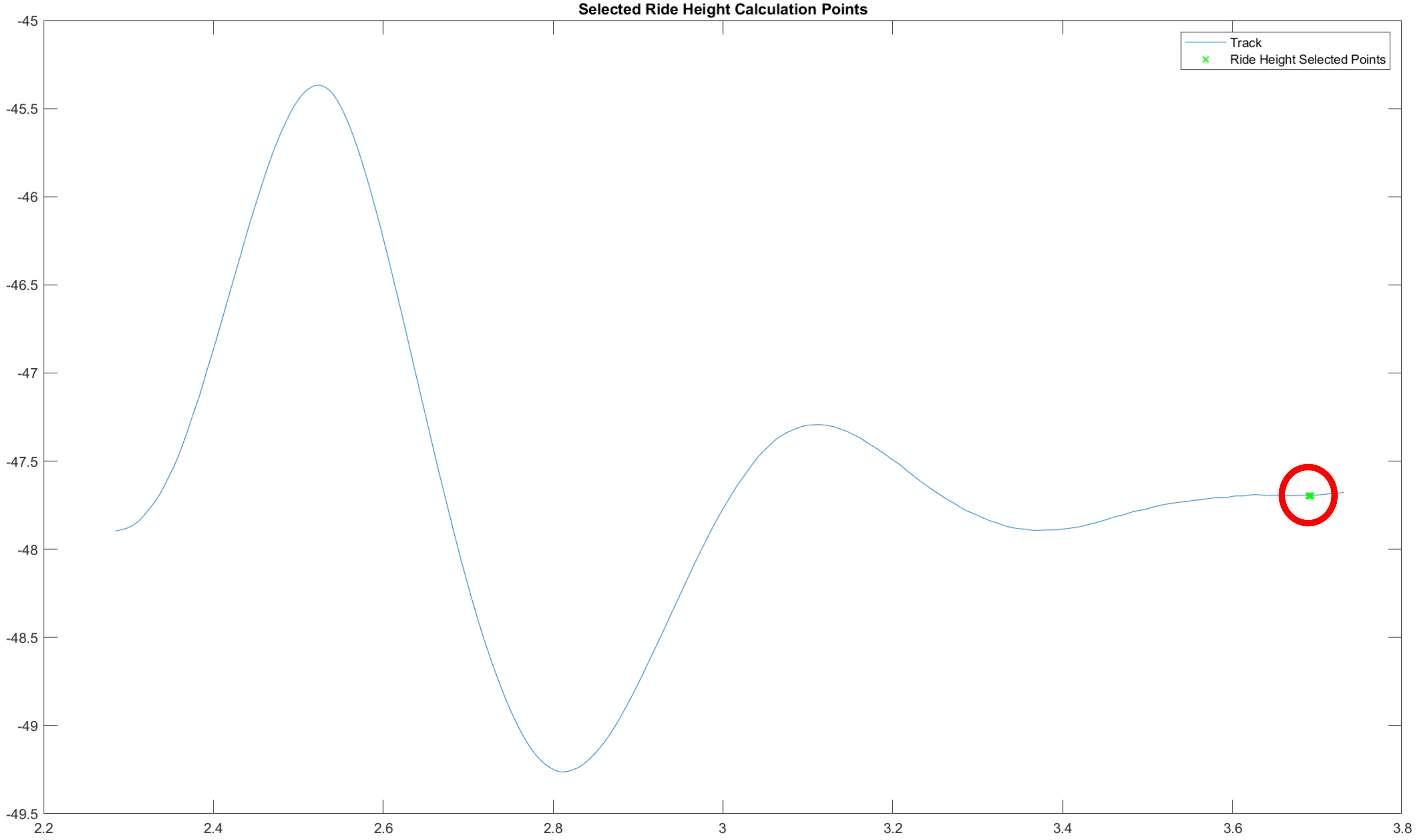
Energy Dissipation Method: Calculating Ride Height

- **In order to calculate ride height, candidate points are identified based on areas where vertical derivative is close to zero**



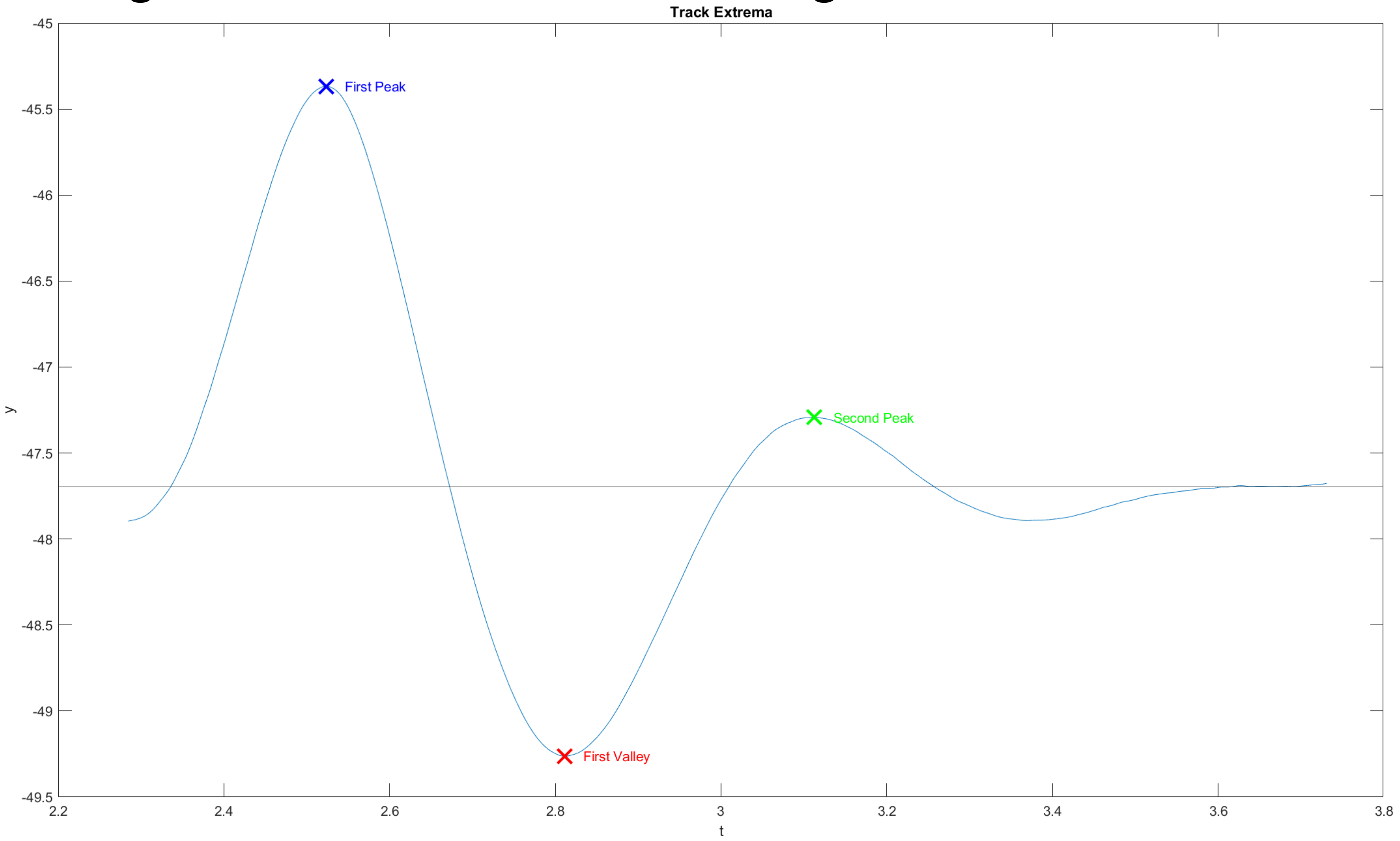
Energy Dissipation Method: Calculating Ride Height

- **Points are selected based on their proximity to the end of the track:**



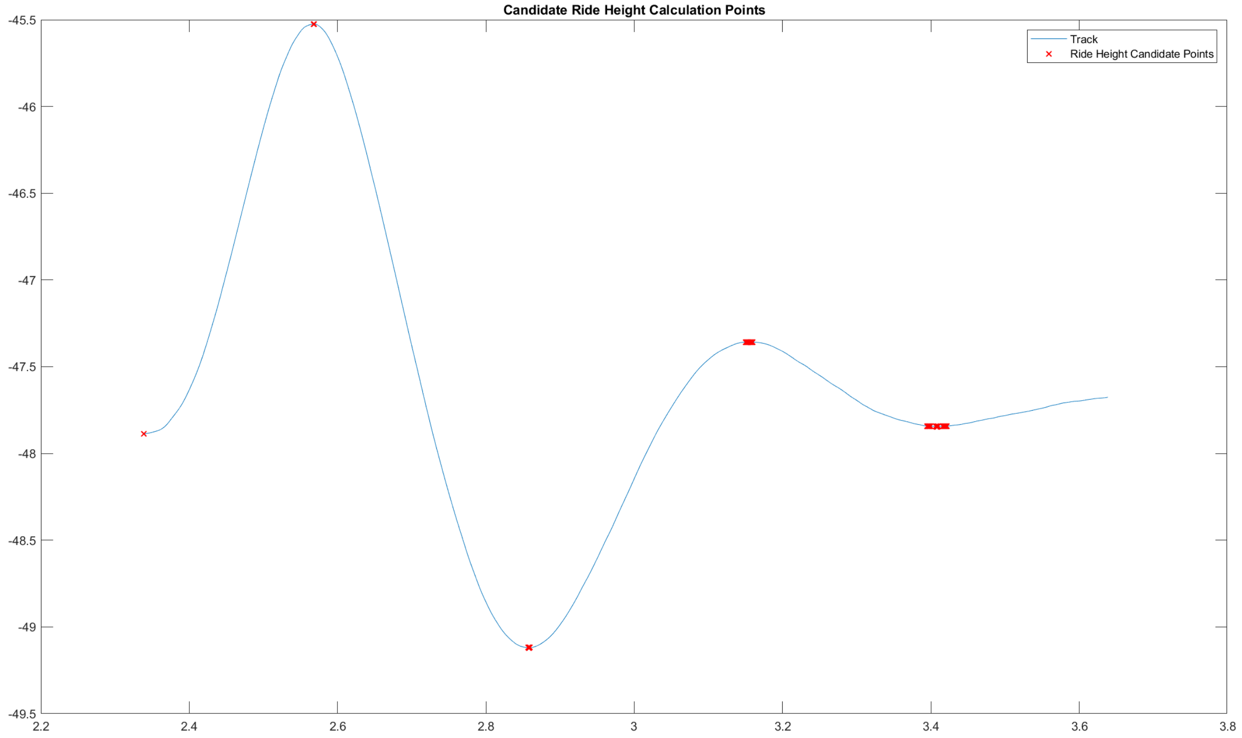
Energy Dissipation Method: Calculating Ride Height

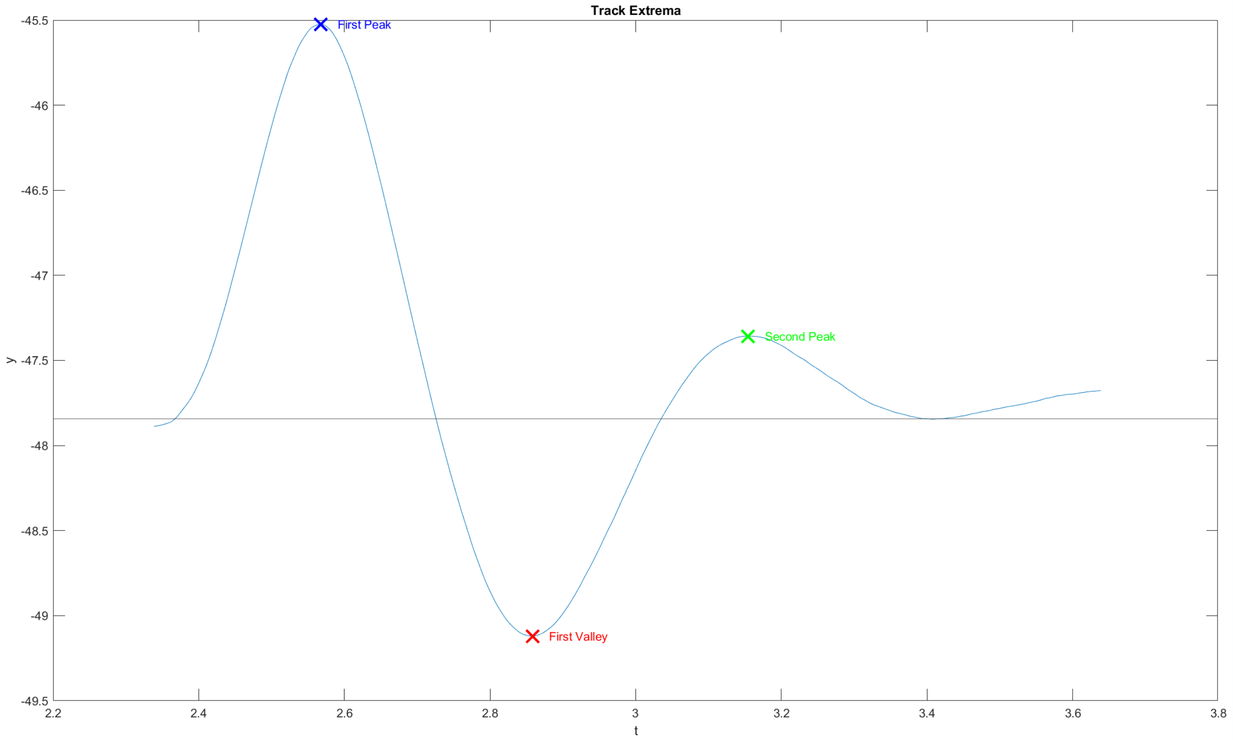
- **Points are averaged and result is used as ride height**



Energy Dissipation Method: Ride Height Issues

- **Method fails when track does not reach equilibrium before end of tracking**
- **Second valley or even second peak can end up being selected as ride height**





Energy Dissipation Method: Ride Height Issues

- **Front right side is even worse since points before the second peak are automatically ignored, leaving no points to average so second peak is used as a fallback**